Globalization with Capital Integration and Spatial Entry*

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Abstract

This paper integrates three features that have not previously been studied jointly: international goods trade, international capital mobility, and heterogeneous-firm entry and exit. Within this unified framework, trade liberalization can generate welfare losses—a sharp departure from standard trade models that typically guarantee gains. The mechanism operates through the interaction between capital mobility and firm heterogeneity, which makes firm entry responsive to factor-price changes.

Calibrating the model to 50 U.S. states, Washington, D.C., and China using a 2017 region-by-region trade-share matrix, I evaluate the effects of President Trump's 2018 tariffs on Chinese imports. The results show that state-level household welfare changes are positively correlated with political preferences, proxied by the Republican vote share in the 2016 presidential election.

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1 Introduction

It is widely believed that trade liberalization generates welfare gains, even when implemented unilaterally. However, globalization policies often face substantial public resistance in practice. From the "giant sucking sound" rhetoric of 1992 to the "Make America Great Again" slogan in 2020, public narratives often portray international trade as a form of economic competition, yet what countries actually compete over remains vague. In contrast, canonical trade models commonly used in the literature leave little room for such cross-country competition.

In a typical Heckscher–Ohlin (H–O) framework, international trade helps countries attract the factors they need. By inducing specialization, trade aligns production with countries' comparative advantage and naturally brings the factors required for that production. In models under the umbrella of new trade theory, where the love of variety drives trade, factors do not move across countries. With factor markets clearing domestically, the overall level of domestic production remains unchanged. The idea of competition arises in Melitz (2003): with heterogeneous firms, trade introduces domestic competition that filters out low-productivity firms and raises the average productivity of surviving firms. However, since factors must still clear domestically, total production remains unchanged, and trade therefore does not generate cross-country competition.

In this paper, I extend the Melitz (2003) model by incorporating an integrated capital market in which capital is required both for production and for paying various fixed costs, including the fixed costs of operating and entering a market. Under this setup, cross-country competition arises for capital. In particular, Section 2 shows that this competition emerges through firms' extensive-margin decisions rather than the intensive margin. Thus, without firm heterogeneity—i.e., under the representative-firm assumption—even with capital mobility, cross-country competition does not arise because firm entry and exit are absent.

Positioning in the literature This work is closely related to the literature studying capital flows in open economies. Beyond the classic Heckscher–Ohlin (H–O) framework, several studies incorporate capital in different ways. Costinot et al. (2014) use a two-country, two-good endowment model in which the allocation of consumption itself implies the assignment of "capital." Lloyd and Marin (2024) extend this setup by introducing trade costs. Hu (2024) adopt a home-bias framework in which cross-country capital investment corresponds to government-issued equity in each country.

The role of capital in this paper follows the traditional assumption that capital is used for production. In this respect, the model is closely related to Edwards and van Wijnbergen (1986), Boulhol (2009), and Kleinman et al. (2023), where capital serves a similar function. Edwards and van Wijnbergen (1986) adopts the Dixit–Norman trade policy framework and assumes a small open economy. Boulhol (2009) build on a Krugman-type model, while Kleinman et al. (2023) employs an Armington structure; both assume homogeneous firms.

This paper is also connected to the literature on firm delocation, including Venables (1985) and Bagwell and Staiger (2012), which analyze delocation in a Cournot competition framework.

Lloyd and Marin (2024) further show that capital control policies can generate welfare gains under certain tax and tariff policies, a result related to the findings of this paper. However, this paper focuses on the welfare consequences of capital mobility without considering tariff revenue.

Another related strand of the literature studies why governments impose trade barriers. Caliendo et al. (2021) extend the Melitz model to incorporate roundabout production and show that firm markups increase production costs, implying that an optimal tariff can correct the resulting distortions. Melitz and Ottaviano (2008) construct a model with quadratic utility that generates linear demand and show that unilateral liberalization can induce firm relocation and lead to welfare losses. While their mechanism is related to the one explored in this paper, their model does not involve capital relocation. This paper instead highlights an alternative channel: governments may impose trade barriers to support domestic firm entry, thereby protecting wage earners by limiting capital outflows.

Finally, this paper is related to research examining the welfare consequences of the China shock. Caliendo et al. (2019) show that wage rigidities can generate short-run welfare losses, while Harrison et al. (2011) demonstrate that welfare gains from trade may be unevenly distributed across heterogeneous agents. In contrast, this paper abstracts from sectoral structural change and focuses on how capital mobility interacts with trade policy to affect welfare outcomes.

2 Two-Country Model

To illustrate the key mechanism and underlying intuition, the analysis begins with a two-country, one-sector model. This minimal setup highlights the core economic forces at work.

2.1 Model Setup

Suppose there are two countries in the world: country J and country L. The total capital endowment in the world is denoted by K_0. Capital ownership is fixed and determined by the exogenous parameter γ: households in country J own a fraction $\gamma \in (0, 1)$ of the world capital stock, while households in country L own the remaining fraction $1 - \gamma$.

Although capital ownership is fixed, households can invest capital in any location without friction, implying perfect capital mobility and a globally equalized rental rate of capital, denoted by r.

2.2 Consumer Sides

Households in both countries share an identical utility function and therefore solve analogous utility-maximization problems. In country $I \in \{J, L\}$, the representative household chooses consumption $q_I(\omega)$ for each variety $\omega \in \Omega_I$, where Ω_I denotes the set of varieties available in country I, to maximize

$$\max_{\{q_I(\omega)\}} U_I = \left(\int_{\omega \in \Omega_I} q_I(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

subject to the budget constraint

$$\int_{\omega \in \Omega_I} p_I(\omega) \, q_I(\omega) \, d\omega = Y_I,$$

where income in country I is given by

$$Y_I = W_I L_I + r K_I.$$

Utility maximization yields the standard CES demand for each variety ω:

$$q_I(\omega) = \frac{p_I(\omega)^{-\sigma}}{P_I^{1-\sigma}} Y_I,$$

where P_I denotes the ideal price index in country I, defined as

$$P_I = \left(\int_{\omega \in \Omega_I} p_I(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}}.$$

2.3 Firm Sides

Each firm produces a differentiated variety ω using a Cobb–Douglas production function form given by

$$q(\omega) = z \left(\frac{l}{\xi} \right)^{\xi} \left(\frac{k}{1-\xi} \right)^{1-\xi},$$

where z denotes firm-level productivity. Firms draw their productivity level z from a country-specific distribution prior to production. After observing z, each firm decides whether to operate, export, or exit the market.

Productivity in country $I \in \{J, L\}$ follows a Pareto distribution with shape parameter

$\alpha_I > \sigma - 1$ and lower bound $\underline{z}^I$, such that

$$\Pr(Z \geq z) = \mathcal{F}^I(z) = \begin{cases} \left(\dfrac{\underline{z}^I}{z}\right)^{\alpha_I}, & z \geq \underline{z}^I, \\ 1, & z < \underline{z}^I. \end{cases}$$

To keep the model as simple as possible, I assume that productivity follows the same Pareto distribution in both countries, so that $\alpha_J = \alpha_L = \alpha$ and $\underline{z}_J = \underline{z}_L = \underline{z}$.

Trade between countries is subject to a source–destination–specific iceberg trade cost, denoted by $\tau_{SI} \geq 0$, so that the consumer price of a variety produced in country S and sold in country I exceeds its producer price by the constant factor $(1+\tau_{SI})$. Since firm productivity z is drawn from a continuous distribution, there is a one-to-one mapping between varieties ω and productivity draws z. Introducing the source-country index S to account for iceberg trade costs does not alter this correspondence. I therefore replace the variety-level notation $p_I(\omega)$ and $q_I(\omega)$ with $p_{SI}(z)$ and $q_{SI}(z)$, respectively.

Given the demand $q_{SI}(z)$ faced in market I, a firm of index (S, z) chooses capital and labor to minimize its production cost. Because serving market I requires shipping $(1 + \tau_{SI})$ units for each unit consumed, the firm must produce $q_{SI}(z)(1 + \tau_{SI})$ units of output.

Cost minimization implies that the amounts of capital and labor required to serve market I are

$$K_{SI}(z) = \frac{1-\xi}{z}\left(\frac{W_S}{r}\right)^{\xi} q_{SI}(z)(1+\tau_{SI}),$$

$$L_{SI}(z) = \frac{\xi}{z}\left(\frac{W_S}{r}\right)^{\xi-1} q_{SI}(z)(1+\tau_{SI}).$$

Under monopolistic competition, firms set prices as a constant markup over marginal cost. Accordingly, the optimal price charged by a firm with productivity z located in country S and selling to country I is

$$p_{SI}(z) = \frac{\sigma}{\sigma-1} W_S^{\xi} r^{1-\xi} \frac{(1+\tau_{SI})}{z}.$$

2.4 Aggregate Values

The value of trade for a variety produced by a firm with productivity z located in country S and sold in country I is given by

$$V_{SI}(z) = p_{SI}(z)\, q_{SI}(z) = \sigma B_I (W_S^{\xi} r^{1-\xi} \frac{1+\tau_{SI}}{z})^{1-\sigma}, \tag{1}$$

where

$$B_I = \frac{W_I L_I + r K_I}{P_I^{1-\sigma}} \left(\frac{1}{\sigma}\right)^{\sigma} \left(\frac{1}{\sigma - 1}\right)^{1-\sigma} \tag{2}$$

I interpret B_I as a measure of a market's *purchase size.* It increases with aggregate income and with the price index. Higher income raises demand for each variety, while a higher price index reflects weaker competition across varieties, which also increases the demand faced by individual firms. As a result, a larger measure of purchase size corresponds to a more favorable market environment for firm survival.

2.5 Technology Threshold for Selling in a Market

A firm located in country S sells to market I only if its variable profits from serving that market are at least as large as the fixed cost of selling there. Formally, a firm with productivity z must satisfy

$$\pi_{SI}(z) \geq F_{SI}\, W_S^{\xi} r^{1-\xi},$$

where F_{SI} denotes the fixed-cost shifter for a firm from country S to sell in market I.

Using the expressions derived above, variable profits from selling to market I are given by

$$\pi_{SI}(z) = V_{SI}(z) - W_S L_{SI}(z) - r K_{SI}(z) = \frac{V_{SI}(z)}{\sigma}.$$

So the productivity cutoff $z_{\min}^{SI}$ for a firm locate at S to sell to market I is thus

$$z_{\min}^{SI} = \left(\frac{F_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}} \left(W_S^{\xi} r^{1-\xi}\right)^{\frac{\sigma}{\sigma-1}} (1+\tau_{SI}). \tag{3}$$

Finally, I define post–fixed-cost profits from selling to market I as

$$\tilde{\pi}_{SI}(z) = \pi_{SI}(z) - F_{SI} W_S^{\xi} r^{1-\xi}.$$

2.6 Aggregated Trade Value

Aggregating across all firms located in country S that sell to market I, let Ω_{SI} denote the set of varieties—equivalently, firms—in country S that serve market I. Total trade value from S to I is then given by

$$V_{SI} = \int_{\omega \in \Omega_{SI}} V_{SI}(\omega)\, d\omega = N_S \Pr(\tilde{\pi}_{SI} > 0)\, \mathbf{E}[V_{SI}(z) \mid \tilde{\pi}_{SI} > 0]\,,$$

where N_S denotes the mass of firms in country S.

Evaluating this expression using the Pareto productivity distribution yields

$$V_{SI} = \frac{\sigma \alpha \underline{z}^{\alpha}}{\alpha + 1 - \sigma} N_S\, B_I^{\frac{\alpha}{\sigma-1}}\, F_{SI}^{1+\frac{\alpha}{1-\sigma}} \left(W_S^{\xi} r^{1-\xi}\right)^{1-\epsilon} (1+\tau_{SI})^{-\alpha}, \tag{4}$$

where $\epsilon \equiv \frac{\sigma\alpha}{\sigma-1}$.

In addition, total post–fixed-cost profits earned by firms from country S selling to market I are given by

$$\tilde{\pi}_{SI} = \int_{\omega \in \Omega_{SI}} \tilde{\pi}_{SI}(\omega)\, d\omega = \int_{\omega \in \Omega_{SI}} \left(\pi_{SI}(\omega) - F_{SI} W_S^{\xi} r^{1-\xi}\right) d\omega. \tag{5}$$

Equation (4) is standard in Melitz-type models with Pareto productivity. Relative to the canonical formulation in Melitz (2003), the only difference is that the wage term W_S is replaced by the composite unit cost $W_S^{\xi} r^{1-\xi}$, reflecting the Cobb–Douglas use of labor and capital in production.

2.7 Equilibrium Conditions

An equilibrium is a collection of wages $\{W_I\}_{I\in\{J,L\}}$, price indices $\{P_I\}_{I\in\{J,L\}}$, and measure of firms $\{N_I\}_{I\in\{J,L\}}$ such that: (i) households maximize utility subject to their budget constraints; (ii) firms choose prices, factor inputs, and market participation to maximize profits; and (iii) labor, capital, and goods markets clear, taking the rental rate of capital r as the numéraire.

Rather than working directly with the price index P_I, I use the term B_I, which is a monotone transformation of P_I, as defined in Equation 2. This transformation does not introduce an additional equilibrium object; it is merely a notational device that simplifies the expression for bilateral trade values V_{SI}.

In equilibrium, the six endogenous variables

$$\{W_J, W_L, N_J, N_L, B_J, B_L\}$$

are jointly determined by free-entry conditions in each country, labor-market–clearing conditions in each country, the global capital-market–clearing condition, and the trade-balance condition in one country (with the corresponding condition in the other country being redundant).

Free entry. For firms located in country J, the free-entry condition requires that total operating profits equal total entry costs:

$$\sum_I \tilde{\pi}_{JI} = N_J F_J^E W_J^{\xi} r^{1-\xi},$$

where F_J^E denotes the fixed-cost shifter for entry in country J, and $\tilde{\pi}_{JI}$ denotes aggregate profits earned by firms from country J in market I after deducting the fixed cost of serving that market as defined in Equation 5. Aggregate post–fixed-cost profits satisfy

$$\sum_I \tilde{\pi}_{JI} = \frac{\sigma - 1}{\sigma\alpha} \sum_I V_{JI}.$$

Substituting this expression into the free-entry condition yields

$$\frac{\sigma - 1}{\sigma\alpha} \sum_I V_{JI} = N_J F_J^E W_J^{\xi} r^{1-\xi}.$$

An analogous free-entry condition holds for firms located in country L:

$$\frac{\sigma - 1}{\sigma\alpha} \sum_I V_{LI} = N_L F_L^E W_L^{\xi} r^{1-\xi}.$$

Labor market clearing. Labor-market–clearing conditions require that the total labor endowment be allocated either to variable production or to the payment of fixed costs.

For firms located in country S, labor is used for three purposes: (i) variable production of goods shipped to destination I, (ii) the fixed cost of selling in market I, and (iii) the fixed cost of entry. Define $\tilde{L}_{SI}$ as the total labor used by firms from country S to serve destination I, including both variable production and fixed selling costs. Specifically,

$$\tilde{L}_{SI} = N_S \int_{\omega \in \Omega_{SI}} \left[L_{SI}(\omega) + \xi F_{SI} \left(\frac{W_S}{r} \right)^{\xi - 1} \right] d\omega = \xi \frac{V_{SI}}{\sigma W_S} \left(\sigma - 1 + \frac{\alpha + 1 - \sigma}{\alpha} \right).$$

Labor-market clearing in country S therefore requires that total labor supply equal labor used for serving all destinations plus labor required for entry:

$$L_S = \sum_I \tilde{L}_{SI} + N_S F_S^E \xi \left(\frac{W_S}{r} \right)^{\xi - 1}.$$

Using the expression for $\tilde{L}_{SI}$, the labor-market–clearing conditions for each country can be written compactly as

$$L_J = \frac{\xi}{W_J}\sum_I V_{JI}, \qquad L_L = \frac{\xi}{W_L}\sum_I V_{LI}.$$

Capital market clearing. With frictionless capital mobility, there is a single integrated capital market. The capital-market–clearing condition requires that the total capital endowment be allocated either to variable production or to the payment of fixed costs.

As with labor, capital is used for three purposes: (i) variable production of goods shipped to destination I, (ii) the fixed cost of selling in market I, and (iii) the fixed cost of entry. Define $\tilde{K}_{JI}$ as the total capital used by firms from country J to serve destination I, including both variable production and fixed selling costs. Analogous definitions apply for firms located in country L.

Under this notation, the aggregate capital-market–clearing condition is

$$K_0 = \sum_I \tilde{K}_{JI} + \sum_I \tilde{K}_{LI} + N_J F_E^J (1-\xi)\left(\frac{W_J}{r}\right)^{\xi} + N_L F_E^L (1-\xi)\left(\frac{W_L}{r}\right)^{\xi}.$$

Note that

$$\sum_I \tilde{K}_{JI} + N_J F_E^J (1-\xi)\left(\frac{W_J}{r}\right)^{\xi}$$

represents the total capital used by firms located in country J. This generally differs from $K_J = \gamma K_0$, which denotes capital ownership. With frictionless capital mobility, ownership and usage need not coincide.

The total capital used by firms from country J to serve destination I is given by

$$\tilde{K}_{JI} = N_J \int_{\omega\in\Omega_{JI}} \left[K_{JI}(\omega) + (1-\xi)F_{JI}\left(\frac{W_J}{r}\right)^{\xi}\right] d\omega = \frac{1-\xi}{\sigma r} V_{JI}\left(\sigma - 1 + \frac{\alpha+1-\sigma}{\alpha}\right).$$

Substituting this expression into the capital-market–clearing condition yields

$$K_0 = \frac{1-\xi}{r} \sum_I (V_{JI} + V_{LI}) \, .$$

Goods market clearing and trade balance. Combining the labor- and capital-market–clearing conditions implies

$$W_J L_J + W_L L_L + r K_0 = \sum_I V_{JI} + \sum_I V_{LI},$$

which states that total expenditure equals total income in the world economy.

Trade-balance conditions require that total expenditure equals total income within each country. For country J,

$$W_J L_J + \gamma r K_0 = V_{JJ} + V_{LJ},$$

and for country L,

$$W_L L_L + (1-\gamma) r K_0 = V_{LL} + V_{JL}.$$

Number of firms. Combining the country-specific free-entry condition with the labor-market–clearing condition yields an expression for the equilibrium mass of firms that is monotonically related to country-specific wages:

$$N_S = \frac{L_S}{\xi F_S^E} \frac{\sigma - 1}{\sigma \alpha} \left(\frac{W_S}{r} \right)^{1-\xi} . \tag{6}$$

Since N_S is a function of W_S, the number of endogenous variables can be reduced from $\{W_J, W_L, B_J, B_L, N_J, N_L\}$ to $\{W_J, W_L, B_J, B_L\}$. Accordingly, the two free-entry conditions can be substituted out when solving for equilibrium.

Summary. The equilibrium is characterized by the following four equations:

$$L_J = \frac{\xi}{W_J} \sum_I V_{JI}, \tag{7}$$

$$L_L = \frac{\xi}{W_L} \sum_I V_{LI}, \tag{8}$$

$$K_0 = \frac{1-\xi}{r} \sum_I \left(V_{JI} + V_{LI}\right), \tag{9}$$

$$\mu_J W_J L_J + \gamma r K_0 = V_{JJ} + V_{LJ}. \tag{10}$$

Here, bilateral sales V_{SI} are obtained by substituting Equation 6 into Equation 4, yielding

$$V_{SI} = \frac{(\sigma-1)\underline{z}^{\alpha}}{\alpha+1-\sigma} \frac{L_S}{\xi F_S^E} \left(\frac{W_S}{r}\right)^{1-\xi} B_I^{\frac{\alpha}{\sigma-1}} F_{SI}^{1+\frac{\alpha}{1-\sigma}} \left(W_S^{\xi} r^{1-\xi}\right)^{1-\epsilon} (1+\tau_{SI})^{-\alpha}. \tag{11}$$

Appendix A solves for the equilibrium under symmetry between the two countries. In the symmetric equilibrium,

$$V_{JL} = V_{LJ}, \qquad V_{JJ} = V_{LL}, \qquad W_J = W_L, \qquad B_J = B_L, \qquad N_J = N_L.$$

This symmetric equilibrium serves as the baseline for the counterfactual analysis.

2.8 Wage–Entry Linkage

Equation 6 shows that the equilibrium number of firms responds to endogenous wage changes, rather than being fully pinned down by exogenous parameters. This wage–entry linkage is novel. In the standard Melitz (2003) framework, the equilibrium mass of firms depends only on labor endowments (e.g., L_I) and model parameters[1]

[1]This linkage does not arise mechanically from introducing capital into production. As shown in Appendix B, when capital is used in production but remains internationally immobile, the equilibrium mass of firms continues to be pinned down by exogenous parameters and is invariant to endogenous wage movements. Appendix C further shows that allowing fixed costs to differ in factor intensities does not alter this result. Finally, Appendix D demonstrates that specifying fixed costs as constants likewise leaves the mechanism unchanged.

The wage–entry linkage emerges specifically from the interaction between capital usage and capital mobility. When firm entry is endogenously determined by the free-entry condition, two general implications arise. First, the number of firms is given by

$$N = \frac{\text{total profit}}{\text{fixed cost}},$$

which in this paper takes the form

$$N_S = \frac{\sum_I \tilde{\pi}_{SI}}{F_S^E W_S^{\xi} r^{1-\xi}}.$$

Second, while the cost function remains homogeneous of degree one in factor prices, it is no longer homogeneous of degree one in quantities due to the presence of fixed costs.

Since factor prices enter the cost function homothetically, and total profit is proportional to cost, the number of firms is determined by relative factor prices, which in this paper are summarized by the wage–rental ratio $\frac{W_I}{r}$.

When all factors are cleared domestically (as in Melitz (2003)), relative factor prices are pinned down by domestic endowments and model parameters. As shown in Appendix B, with two immobile factors,

$$\frac{r_S}{W_S} = \frac{1-\xi}{\xi} \frac{L_S}{K_S}, \tag{12}$$

where ξ, K_S, and L_S are all exogenous.

However, when one factor is mobile, the two factor prices are no longer perfectly linked, such that

$$\frac{r}{W_S} = \frac{1-\xi}{\xi} \frac{L_S}{K_0} \frac{\sum_S \sum_I V_{SI}}{\sum_I V_{SI}}. \tag{13}$$

Here, ξ, K_0, and L_S are exogenous parameters, while $\frac{\sum_I V_{SI}}{\sum_S \sum_I V_{SI}}$ is endogenously determined and represents country S's share of world revenue. From the labor market clearing conditions

(Equations 7 and 8), this share can be written as

$$\frac{\sum_I V_{SI}}{\sum_S \sum_I V_{SI}} = \frac{W_S L_S}{\sum_S W_S L_S}. \tag{14}$$

In the symmetric two-country case, this expression reduces to

$$\frac{r}{W_J} = \frac{1-\xi}{\xi} \frac{L_S}{K_0} \left(1 + \frac{W_L}{W_J}\right). \tag{15}$$

What does Equation (15) imply? First, since

$$1 + \frac{W_L}{W_J} > 1,$$

the rental–wage ratio under capital mobility is strictly higher than in the benchmark economy in which both factors are immobile. Capital mobility therefore raises the relative price of capital and dampens wages relative to the immobile-factor economy.

More importantly, Equation (15) shows that the rental–wage ratio is decreasing in country J's wage relative to country L's wage. When W_J/W_L increases, country J captures a larger share of world revenue, which lowers its relative rental price. Intuitively, higher wages are associated with a larger domestic market and higher GDP, which attracts a greater share of mobile capital for production in country J, regardless of capital ownership. This increase in effective capital availability reduces the relative rental rate, making firm entry cheaper and leading to a larger equilibrium mass of firms in country J.

2.9 Comparative Statics

Given the symmetric baseline solution, I study comparative statics with respect to changes in iceberg trade costs. My primary goal is to analyze how an exogenous change in trade costs influences equilibrium outcomes.

2.9.1 Log-differentiating the trade values

Log-differentiating Equation 6 yields

$$\hat{N}_S = (1 - \xi)\hat{W}_S. \tag{16}$$

Thus, changes in the mass of firms are positively correlated with changes in wages.

At the same time, log-differentiating the trade value equation (Equation 4) yields

$$\hat{V}_{SI} = \hat{N}_S + (1 - \epsilon)\xi\hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha\hat{\tau}_{SI}. \tag{17}$$

Substituting Equation 16 into Equation 17 gives

$$\hat{V}_{SI} = (1 - \xi)\hat{W}_S + (1 - \epsilon)\xi\hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha\hat{\tau}_{SI}.$$

Collecting terms in $\hat{W}_S$ yields

$$\hat{V}_{SI} = (1 - \xi\epsilon)\hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha\hat{\tau}_{SI}. \tag{18}$$

The elasticity of sales with respect to wages in response to a tariff change is therefore $1 - \xi\epsilon$, which can be positive. In contrast, in canonical trade models this elasticity is always negative.[2]

2.9.2 Log-differentiating the equilibrium conditions

Given the generalized log-differentiated expression for sales revenue, I now log-differentiate the equilibrium conditions.

[2] See Appendix E for further discussion of the differences.

Labor market clearing conditions. Log-differentiating the labor market clearing conditions yields:

$$\sum_I V_{SI}\, \hat{W}_S = \sum_I V_{SI}\, \hat{V}_{SI}, \tag{19}$$

Substituting Equation 18 into the log-linearized labor market clearing conditions above gives:

$$\sum_I V_{SI} \hat{W}_S = (1 - \xi\epsilon) \sum_I V_{SI} \hat{W}_S + \frac{\alpha}{\sigma - 1} \sum_I V_{SI} \hat{B}_I - \alpha \sum_I V_{SI} \hat{\tau}_{SI}, \tag{20}$$

Capital market clearing conditions. Log-linearizing the capital market clearing condition in Equation 9 yields:

$$\sum_I V_{JI} \hat{V}_{JI} + \sum_I V_{LI} \hat{V}_{LI} = 0. \tag{21}$$

Using Equations 19, Equation 21 can be rewritten as:

$$\sum_I V_{JI} \hat{W}_J + \sum_I V_{LI} \hat{W}_L = 0. \tag{22}$$

The capital market clearing condition therefore implies that equilibrium wage changes in the two countries must move in opposite directions; in the symmetric-country case, they move in opposite directions with equal magnitude from the baseline equilibrium.

Trade balance conditions. Log-differentiating the trade balance condition for country J yields:

$$W_J L_J\, \hat{W}_J = V_{JJ} \hat{V}_{JJ} + V_{LJ} \hat{V}_{LJ}.$$

Substituting Equation 18 into this expression gives:

$$\big(\xi \sum_I V_{JI} + V_{JJ}(\xi\epsilon - 1)\big) \hat{W}_J + (\xi\epsilon - 1) V_{LJ} \hat{W}_L + \frac{\alpha(V_{JJ} + V_{LJ})}{1 - \sigma} \hat{B}_J = -\alpha V_{LJ} \hat{\tau}_{LJ}. \tag{23}$$

Matrix representation. The log-linearized equilibrium conditions can be summarized in the following general matrix form. In this representation, $\hat{N}_J$ and $\hat{N}_L$ have been substituted out using Equations 16, so that the system is expressed solely in terms of $\hat{W}_J$, $\hat{W}_L$, and $\hat{B}_J$, $\hat{B}_L$.[3]

$$\begin{pmatrix} \xi\epsilon\sum_I V_{JI} & 0 & \frac{\alpha}{1-\sigma}V_{JJ} & \frac{\alpha}{1-\sigma}V_{JL} \\ 0 & \xi\epsilon\sum_I V_{LI} & \frac{\alpha}{1-\sigma}V_{LJ} & \frac{\alpha}{1-\sigma}V_{LL} \\ \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\ \xi\sum_I V_{JI} + (\xi\epsilon - 1)V_{JJ} & (\xi\epsilon - 1)V_{LJ} & \frac{\alpha}{1-\sigma}(V_{JJ} + V_{LJ}) & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha V_{JL}\hat{\tau}_{JL} \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \end{pmatrix}$$

I further simplify the expression by defining

$$\lambda \equiv \frac{V_{JL}}{V_{JL} + V_{JJ}},$$

which represents the share of country J's GDP derived from exports. Using the symmetry of the equilibrium, this expression can be rewritten as

$$\begin{pmatrix} \xi\epsilon & 0 & \frac{\alpha(1-\lambda)}{1-\sigma} & \frac{\alpha}{1-\sigma}\lambda \\ 0 & \xi\epsilon & \frac{\alpha\lambda}{1-\sigma} & \frac{\alpha}{1-\sigma}(1-\lambda) \\ 1 & 1 & 0 & 0 \\ \xi + (\xi\epsilon - 1)(1-\lambda) & (\xi\epsilon - 1)\lambda & \frac{\alpha}{1-\sigma} & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha\lambda\hat{\tau}_{JL} \\ -\alpha\lambda\hat{\tau}_{LJ} \\ 0 \\ -\alpha\lambda\hat{\tau}_{LJ} \end{pmatrix}$$

[3] A version of the matrix system that explicitly includes $\hat{N}_J$ and $\hat{N}_L$ is provided in Appendix F.

2.9.3 Bilateral trade liberalization

I first consider a symmetric bilateral trade liberalization in which both countries reduce bilateral trade costs by the same amount,

$$\hat{\tau}_{JL} = \hat{\tau}_{LJ} \equiv \hat{\tau} < 0.$$

Solving the system of log-linearized equilibrium conditions yields

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \lambda(\sigma-1)\hat{\tau} \\ \lambda(\sigma-1)\hat{\tau} \end{pmatrix}.$$

A bilateral trade liberalization therefore does not affect wages in either country relative to the rental rate of capital. As a result, nominal household income remains unchanged in both countries. However, the trade liberalization lowers the price index through changes in B_J and B_L, and this price index adjustment is identical across countries under symmetry.

Household welfare changes in country $I \in \{J, L\}$ are given by changes in real income,

$$(\frac{\hat{W}_I W L + \hat{r} r K_I}{W_I L_I + r K_I}) - \hat{P}_I.$$

Since the rental rate is the numeraire, $\hat{r} = 0$, so welfare changes reduce to

$$\frac{W_I L_I}{W_I L_I + r K_I} \hat{W}_I - \hat{P}_I = s_I \hat{W}_I - \hat{P}_I.$$

Here, s denotes the labor income share,

$$s_I \equiv \frac{W_I L_I}{W_I L_I + r K_I},$$

which equals ξ in the symmetric benchmark.

Proposition 1 *Under bilateral trade liberalization, real income in each country $I \in \{J, L\}$ increases by the same amount:*

$$s_I \hat{W}_I - \hat{P}_I = -\hat{P}_I = -\frac{1}{\sigma - 1}\big(\hat{B}_I - s_I \hat{W}_I\big) = -\lambda\hat{\tau} > 0.$$

2.9.4 Unilateral trade liberalization

Consider a unilateral trade liberalization in which only country J reduces the trade barrier applied to imports from country L. That is, the trade cost for goods shipped from L to J falls, while the trade cost from J to L remains unchanged:

$$\hat{\tau}_{JL} = 0, \qquad \hat{\tau}_{LJ} = \hat{\tau} < 0.$$

Solving the system of log-linearized equilibrium conditions yields

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} \frac{\alpha\lambda(1-\lambda)}{A}\,\hat{\tau} \\ -\frac{\alpha\lambda(1-\lambda)}{A}\,\hat{\tau} \\ (\sigma-1)\frac{\lambda C}{A}\,\hat{\tau} \\ -(\sigma-1)\frac{(1-\lambda)D}{A}\,\hat{\tau} \end{pmatrix}.$$

where

$$A = (1-\xi) + 2\lambda\xi + 4\lambda(1-\lambda)(\epsilon\xi - 1) > 0,$$

$$C = \lambda\big(\xi - (1-2\lambda)(1-\epsilon\xi)\big) + \epsilon\xi,$$

$$D = \lambda\big(\xi - (1-2\lambda)(1-\epsilon\xi)\big).$$

Proposition 2 *Under unilateral trade liberalization, if the export share of GDP satisfies $\lambda < \frac{1}{2}$, nominal wages in country J decrease, while nominal wages in country L increase by* *the same magnitude.*

Proof. Since $\hat{\tau} < 0$, the sign of the wage response in country J is determined by the sign of the coefficient $\alpha\lambda(1-\lambda)/A$. For $\lambda \in (0,1)$, the numerator is strictly positive. As shown in Appendix G, the denominator A is strictly positive when $\lambda < \frac{1}{2}$, a condition that is empirically plausible. Therefore, $\hat{W}_J < 0$. By the capital market clearing condition, equilibrium wage changes must have equal magnitude and opposite signs, implying $\hat{W}_L = -\hat{W}_J > 0$. ■

Implications for GDP. Using Proposition 2, we now characterize the change in GDP for country J under unilateral trade liberalization. Let total GDP in country J be the value of domestic sales plus exports. Log-differentiating yields

$$\hat{V}_J = (1-\lambda)\hat{V}_{JJ} + \lambda\hat{V}_{JL}. \tag{24}$$

Substituting Equation 18 into this expression gives

$$\hat{V}_J = (1-\lambda)(1-\xi\epsilon)\hat{W}_J + \frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J + \lambda(1-\xi\epsilon)\hat{W}_L + \frac{\alpha\lambda}{\sigma-1}\hat{B}_L. \tag{25}$$

Using the fact that $\hat{W}_J = -\hat{W}_L$, this expression can be rewritten as

$$\hat{V}_J = (1-\xi\epsilon)\hat{W}_J + \frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J + \frac{\alpha\lambda}{\sigma-1}\hat{B}_L. \tag{26}$$

Substituting the closed-form solutions for $\hat{B}_J$ and $\hat{B}_L$ shows that

$$\frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J = C\,\hat{W}_J, \qquad \frac{\alpha\lambda}{\sigma-1}\hat{B}_L = -D\,\hat{W}_J,$$

Equation 26 therefore simplifies to

$$\hat{V}_J = \big[(1-\xi\epsilon) + (C-D)\big]\hat{W}_J, \tag{27}$$

which reduces to

$$\hat{V}_J = \hat{W}_J. \tag{28}$$

Given that $\hat{W}_J$ increases with an increase in $\hat{\tau}_{LJ}$, a unilateral increase in import tariffs raises domestic GDP in country J.

Moreover, an increase in GDP in country J implies a higher demand for capital. Since capital used in country J satisfies

$$\tilde{K}_J = (1 - \xi) V_J,$$

it follows that

$$\hat{\tilde{K}}_J = \hat{V}_J.$$

Therefore, a unilateral tariff increase induces a reallocation of capital toward country J, drawing capital away from the foreign country. In Appendix B, Paragraph B.6, I show that, in the absence of capital mobility, domestic GDP is invariant to trade-cost changes of any kind. The endogenous response of domestic GDP is thus the central mechanism generating the distinct outcomes in the present framework.

Implications for the Price Index Substituting the solution into the log-change expression for the price index,

$$\hat{P}_I = \frac{1}{\sigma - 1} \left(\hat{B}_I - \xi \hat{W}_I \right), \tag{29}$$

yields

$$\hat{P}_J = \frac{(\lambda + \sigma - 1)\hat{V}_J - (\lambda + \sigma - 1)\hat{N}_J - \lambda \hat{B}_L}{(\sigma - 1)(1 - \lambda)}. \tag{30}$$

For comparison, in the benchmark case with immobile capital, the domestic price index reduces to

$$\hat{P}_J = -\frac{\lambda}{1 - \lambda} \hat{B}_L, \tag{31}$$

as shown in Equation 81. In this case, the domestic price index depends solely on foreign "purchasing power".

With capital mobility, the domestic price index is no longer a function only of foreign "purchasing power." The presence of the factor $\frac{1}{\sigma-1}$ further implies that the influence of foreign "purchasing power" is attenuated. In addition to the term $\hat{B}_L$, the price index depends on domestic aggregate production and the domestic mass of firms. The coefficient on $\hat{V}_J$ is positive, consistent with higher domestic production being associated with higher equilibrium wages and production costs, and thus a higher price index. In contrast, the coefficient on $\hat{N}_J$ is negative, indicating that an increase in the number of domestic firms lowers the price index, as consumers substitute toward a larger set of domestically produced varieties that are not subject to iceberg trade costs. As a result, increased firm entry partially offsets the price-increasing effect of higher domestic output. Overall, the elasticity of the domestic price index with respect to a unilateral tariff change is smaller than in the case of immobile capital. Under normal circumstances, a unilateral tariff increase reduces foreign "purchasing power" and therefore raises the domestic price index; however, in the present model this price effect is quantitatively weaker.

An abnormal case may also arise in this model because the term $1 - \epsilon\xi$ is not necessarily negative. If $1 - \epsilon\xi > 0$ and its magnitude is sufficiently large—an outcome that is extremely rare and occurs only for very small values of ξ (e.g., $\xi < 0.05$)—foreign "purchasing power" may increase following a unilateral tariff increase. The underlying mechanism is that a sufficiently large capital inflow into the domestic country J leads to a sharp decline in the foreign mass of firms, forcing foreign consumers to substitute toward goods produced in country J. In this case, the domestic price index may decrease in response to a unilateral tariff increase.

I now further characterize the conditions under which the abnormal case arises. Substituting the solution into the expression for the change in the domestic price index yields

$$\hat{P}_I = \frac{\lambda}{A}\left(C - \xi\frac{\alpha(1-\lambda)}{\sigma-1}\right)\hat{\tau} = \frac{\lambda}{A}F\hat{\tau}. \tag{32}$$

When $F > 0$, a unilateral tariff reduction leads to a decrease in the domestic price index, generating welfare gains for investors. In contrast, when $F < 0$, the domestic price index increases following a tariff reduction, as in the Metzler paradox, and investors experience welfare losses.

The condition for $F < 0$ is equivalent to

$$\lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big)+\xi\frac{\alpha(1-\lambda)}{\sigma-1} > \epsilon\xi. \tag{33}$$

where

$$F = C - \xi\frac{\alpha(1-\lambda)}{\sigma-1},$$

Since $\xi\frac{\alpha(1-\lambda)}{\sigma-1} > 0$, a sufficient condition for $F < 0$ is $C < 0$, which gives

$$C < 0 \iff \lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big) > \epsilon\xi. \tag{34}$$

By continuity, the left-hand side of inequality (34) is decreasing in ξ. By the Intermediate Value Theorem, there exists a unique threshold $\xi^* \in (0,1)$ such that the inequality holds with equality. It follows that $C < 0$ if and only if $\xi < \xi^*$. Since $C < 0$ is a sufficient condition for $F < 0$, there therefore exist values of ξ for which $F < 0$ holds.

In general, smaller values of ϵ and ξ make it more likely that $F < 0$, implying a greater likelihood that the domestic price index increases in response to a unilateral tariff reduction.

Welfare Implications The change in household welfare is given by

$$\xi\hat{W}_I - \hat{P}_I = \xi\left(1+\frac{\xi}{\sigma-1}\right)\hat{W}_I - \frac{\hat{B}_I}{\sigma-1}.$$

Households experience welfare losses from unilateral trade liberalization if the following condition is satisfied:

$$\xi\left(1+\frac{\xi}{\sigma-1}\right)\alpha(1-\lambda)+\lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big)>\epsilon\xi. \tag{35}$$

Since $\alpha(1-\lambda)>0$, Condition 35 is less restrictive than Condition 33. Consequently, the set of parameter values satisfying Condition 35 strictly contains those satisfying Condition 33. It follows that there exist two thresholds $\bar{\xi}_1<\bar{\xi}_2$ such that, for all $\xi\in(\bar{\xi}_1,\bar{\xi}_2)$, the domestic price index decreases under trade liberalization while households still experience welfare losses.

Export Share of GDP (λ) Implication Finally, I want to analytically discuss the change of export share of GDP, $\hat{\lambda}$ under the unilateral tariff change. Log-differentiating $\lambda=\frac{V_{JL}}{V_{JJ}+V_{JL}}$ under unilateral trade liberalization (i.e., $\hat{\tau}_{JL}=0$) gives:

$$\hat{\lambda}=(1-\lambda)(\hat{V}_{JL}-\hat{V}_{JJ})=(1-\lambda)\frac{\alpha}{\sigma-1}(\hat{B}_L-\hat{B}_J) \tag{36}$$

Substituting $\hat{V}_{SI}$ from Equation 11, I obtain:

$$\hat{\lambda}=-\frac{\alpha\lambda(1-\lambda)\hat{\tau}}{A}(\xi-(1-2\lambda)(1-\epsilon\xi)+\epsilon\xi). \tag{37}$$

The condition for $\hat{\lambda}$ to increase with a unilateral tariff increase (or decrease with a unilateral trade liberalization increase) is thus:

$$(1-2\lambda)(1-\epsilon\xi)-\xi>\epsilon\xi. \tag{38}$$

Compare the condition 38 with condition 34, one can clearly see that condition 34 implies condition 38, however, the ordering between condition 35 and condition 38 is not very obvious. In the following proposition, I proved that there is actually a clear order between those conditions.

Proposition 3 *Condition 33* $\Rightarrow$ *Condition 38* $\Rightarrow$ *Condition 35*

Proof. Proof can be found in Appendix H. ■

Proposition 3 shows that households may experience welfare losses from unilateral trade liberalization even when such a policy increases the export share of GDP and lowers the domestic price index. More concretely, even if the data indicate that domestic firms are exporting more to foreign markets and consumers face lower prices, households can still suffer welfare losses.

3 Multi-Region Calibration

With the intution from a two-country framework in-hand, this section extends the two-country environment to an n-region setting.

3.1 Equilibrium Conditions

Consider n regions indexed by $S \in \{1, \ldots, n\}$. Each region S is endowed with labor L_S and a share γ_S of the aggregate capital endowment K_0, where $\sum_{S=1}^{n} \gamma_S = 1$. Let V_{SI} denote total sales in destination I by firms based in region S. The equilibrium is characterized by the following conditions.

n **Free entry conditions.** For each origin region S, the free-entry condition is

$$\frac{\sigma - 1}{\sigma \alpha_S} \sum_{I=1}^{n} V_{SI} = N_S F_S^E W_S^{\xi} r^{1-\xi}. \tag{39}$$

n **Labor-market clearing conditions.** For each region S,

$$L_S = \frac{\xi}{W_S} \sum_{I=1}^{n} V_{SI}. \tag{40}$$

$n-1$ **Trade balance conditions.** For each region S, imposed for $n-1$ regions,

$$W_S L_S + \gamma_S r K_0 = \sum_{I=1}^{n} V_{IS}. \tag{41}$$

one Global capital-market clearing condition. Aggregate capital demand equals the world capital endowment:

$$K_0 = \frac{1-\xi}{r} \sum_{S=1}^{n} \sum_{I=1}^{n} V_{SI}. \tag{42}$$

3.2 Calibration

The parameters calibrated in the multi-region model are

$$\Theta = \Big\{\sigma,\ \xi_S,\ \gamma_S,\ F_{e,S},\ \underline{z}_S,\ \alpha_S \in \mathbb{R}^{52};\ F_{SI},\ \tau_{SI} \in \mathbb{R}^{52\times 52}\Big\}.$$

For the elasticity of substitution, σ, I follow the empirical estimates in Broda and Weinstein (2006) and Hottman et al. (2016) and set $\sigma = 3$. These estimates are obtained from trade models that are not necessarily formulated within a Melitz (2003)-type heterogeneous-firm framework.

An alternative strand of the literature focuses explicitly on the Pareto-distributed firm productivity assumption in Chaney (2008) and estimates the firm-size distribution directly. Studies such as Akgul and Ahmad (2020) and Nigai (2017) find a Pareto shape parameter close to $\alpha = 2$. Since convergence of aggregate output in the Melitz model requires $\alpha > \sigma - 1$, this restriction implies that σ must be smaller than 3. The closest integer value consistent with this condition is therefore $\sigma = 2$.

In the robustness analysis, I show that recalibrating the model with $\sigma = 2$ yields quantitatively similar results, indicating that the main findings are not sensitive to the precise choice of σ within this empirically relevant range.

I assume the following parameters to be common across regions:

$$\{\xi,\ F_e,\ \underline{z},\ \alpha\}.$$

[4] I follow empirically validated values for ξ and commonly used calibrations in the literature for the remaining parameters.

Parameter	**Value**
σ	3
α	3
$\underline{Z}$	1
ξ	0.6
K_0	1
F^E	1

Table 1: Parameter values for the calibration.

To calibrate the 52×52 matrix of bilateral fixed trade costs F_{SI}, I use trade data for 51 U.S. regions (the 50 states and Washington, D.C.) and China for the year 2017. State-by-state domestic trade flows within the United States are obtained from the Freight Analysis Framework (FAF),[5] while state-level exports to and imports from China are constructed using data from USA Trade Online.[6]

China's domestic sales are not directly observed in the data. I therefore impute China's domestic sales using relative GDP sizes. Specifically, I first compute total U.S. sales (including both domestic sales and exports to China). I then scale this aggregate by the

[4]While these parameters could in principle be calibrated at the region level, doing so would substantially complicate the calibration procedure. Moreover, preliminary region-specific estimates exhibit little cross-region variation.

[5]U.S. Department of Transportation, Bureau of Transportation Statistics (BTS), and Federal Highway Administration (FHWA), *Freight Analysis Framework (FAF), Version 5*, 2017–2050. DOI: 10.21949/1529116.

[6]U.S. Census Bureau, *USA Trade Online*, https://usatrade.census.gov/.

U.S.–China GDP ratio to obtain an estimate of China's total sales. Finally, I subtract total Chinese exports to the United States from this estimate to back out China's domestic sales. Normalizing the resulting bilateral trade flow matrix by total global sales then yields the empirical trade-share moments used in the calibration.

For bilateral iceberg trade costs (tariffs) τ_{SI}, I assume that there are no tariffs on trade within the United States. For U.S. exports to China, I use the average Chinese tariff on U.S. goods in 2017 (0.0383) and assign this value uniformly across U.S. states. Similarly, for Chinese exports to the United States, I use the average U.S. tariff applied to Chinese goods in 2017 (0.0166).

Finally, for the region-specific capital shares γ_S, the equilibrium conditions imply

$$\gamma_S = \frac{\sum_S V_{SI} - \xi \sum_I V_{SI}}{(1-\xi) \sum_S \sum_I V_{SI}}. \tag{43}$$

Thus, γ_S can be computed directly from the trade matrix using the row sums, column sums, and the sum of all entries. Alternatively, one can compute γ_S using regional GDP, regional personal consumption expenditure shares, and aggregate GDP across all regions, since row sums proxy for regional GDP, column sums proxy for personal consumption expenditures, and the sum of all entries proxies for total GDP. The resulting values of γ_S are very similar across the two approaches.

Procedurally, I first fix the parameter values listed in Table 1 and compute γ_S directly from the aggregated trade matrix data. Conditional on these parameters, I estimate the bilateral trade cost matrix F_{SI} by nonlinear least squares (equivalently, equally weighted GMM), targeting the 52×52 matrix of observed trade-share moments.

3.3 Calibration Results

A visualization of the calibrated F_{SI} matrix is provided in Appendix I. The model matches the targeted moments closely. Figure 1 plots the flattened 52×52 matrix of model-implied trade-share moments against their data counterparts, with each point representing a single matrix entry. The fitted line closely follows the 45° line, indicating a tight correspondence between the model-implied and target moments.

As additional validation, I compare model-implied regional aggregates with BEA regional data. Appendix I shows that the model-implied GDP, PCE, and wage shares closely match their BEA-reported counterparts.

To assess the model's performance on non-targeted moments, I examine its ability to reproduce the regional distribution of firms. Although the number of firms is a key endogenous outcome of the model, it is not directly targeted in the calibration. I construct regional firm shares using state-level establishment counts from the County Business Patterns (CBP) and compare them with their model-implied counterparts.

Figure 2 reports the corresponding comparison for the *non-targeted* number-of-establishments share moments. The model exhibits a strong positive relationship between model-implied and data firm shares across regions, indicating that it captures the cross-regional distribution of firms reasonably well despite not being explicitly calibrated to this dimension. Deviations from the 45-degree line arise from the assumption of a region-invariant fixed operating cost F_e, which compresses cross-regional variation in firm size and entry.

4 2018 Tariff Policy

Using the model calibrated in the previous section, I analyze the welfare consequences of President Trump's 2018 tariff policy toward China. I model the policy as an increase in tariffs imposed by the 51 U.S. regions on imports from China, from an average of 1.66% to 25%. Results that additionally account for Chinese retaliation—modeled as a symmetric increase in China's tariffs on U.S. regions to 25%—are reported in the Appendix. Chinese retaliation reduces the welfare gains from the tariff policy, but the magnitude of this effect is relatively small.

Figure 3 shows the percentage change of household welfare resulting from a unilateral tariff increase by the United States. The analysis didn't assumes anything about s which is the labor income share, it directly used the change of total consumption expenditure as the change of personal income because they are the same under the static model setup.

Theoretically speaking, with the rental rate normalized as the numeraire, the larger the share of rental income in household income, the more household welfare is driven by changes in the price index (negatively). Households that rely entirely on rental income do not benefit from wage changes; their welfare is determined solely by movements in the price index. Figure 4 reports percentage changes in real wages, providing a reference for evaluating welfare outcomes under alternative household income compositions.

I validate the welfare changes predicted by my model using BEA-reported regional real personal income (RPI) and real personal consumption expenditure (RPCE). In my static model, the trade balance condition implies that personal income is equivalent to personal consumption expenditure, and welfare changes are therefore measured by the percentage change in real personal income (equivalently, real personal expenditure). In the data, however, these two measures are not equivalent, and household welfare is affected by many factors beyond tariff policy alone.

To discipline the validation exercise, I adopt a simple measurement framework. Suppose there exists a true change in household welfare, denoted by $\hat{\text{welfare}}^*$, which can be decomposed into a component driven by tariff policy and a component driven by all other factors:

$$\hat{\text{welfare}}^*_S = \hat{\text{welfare}}^*_{\text{tariff},S} + \hat{\text{welfare}}^*_{\text{other},S} \tag{44}$$

The welfare change predicted by my model is a noisy indicator of the tariff-driven component, such that

$$\hat{\text{welfare}}^{\text{model}}_S = \mu \hat{\text{welfare}}^*_{\text{tariff},S} + \epsilon_S \tag{45}$$

where ϵ captures model mis-specification and aggregation error.

The data-reported real personal income and real personal consumption expenditures are noisy indicators of the true welfare change. In particular,

$$\widehat{\mathrm{RPCE}}_S = \alpha_1 \widehat{\mathrm{welfare}}^*_{\mathrm{tariff},S} + \alpha_2 \widehat{\mathrm{welfare}}^*_{\mathrm{other},S} + \nu_S \tag{46}$$

$$\widehat{\mathrm{RPI}}_S = \beta_1 \widehat{\mathrm{welfare}}^*_{\mathrm{tariff},S} + \beta_2 \widehat{\mathrm{welfare}}^*_{\mathrm{other},S} + \iota_S \tag{47}$$

With two noisy signals, a linear combination of $\widehat{\mathrm{RPCE}}$ and $\widehat{\mathrm{RPI}}$ can weakly improve the proxy for $\hat{W}^{\mathrm{tariff}}$ relative to using either measure alone, by partially filtering out the non-tariff component $\hat{W}^{\mathrm{other}}$.

The first column of Table 2 provides evidence consistent with the idea that combining the two indicators helps attenuate welfare changes driven by non-tariff factors, and that the model-predicted tariff-induced welfare change aligns with the corresponding data proxy.

In the second column, I evaluate the performance of the original Melitz (2003) framework, which abstracts from capital and capital mobility.[7] As in the first column, the coefficients on RPCE and RPI display a mirror pattern, with similar magnitudes and opposite signs. However, unlike the results from my model, both coefficients are statistically insignificant. This indicates that the original Melitz (2003) framework does not approximate tariff-induced welfare changes as effectively as the model with capital and capital mobility.

The third and fourth columns report placebo tests based on pre-policy variation. Specifically, I replace the 2017–2018 changes in RPCE and RPI with their percentage changes from 2016 to 2017, a period preceding the implementation of the tariff policy. Because no major tariff shocks occurred during this period, the model-predicted tariff-induced welfare change should not systematically align with these pre-policy income or expenditure changes. Consistent with this expectation, the coefficients in Columns (3) and (4) are statistically insignificant, providing further support that the significant results in Column (1) reflect the tariff policy shock rather than spurious correlations.

[7] I use the same calibrated parameters for this specification, except that labor intensity is set to $\xi = 1$ and the regional capital endowment shares γ_S are not used when computing the equilibrium.

In the fifth column, I conduct a horse-race regression that jointly includes the welfare changes predicted by my model and by the Melitz (2003) framework. The coefficient on my model-predicted welfare change is positive and statistically significant, whereas the coefficient on the Melitz-predicted welfare change is small and statistically insignificant. This result indicates that, conditional on both measures, variation in regional real personal income changes is primarily explained by the welfare changes predicted by the model with capital and capital mobility[8].

5 Discussion

In this section, I examine the relationship between the model-predicted welfare change and the Republican vote share in the 2016 election. Figure 5 suggests that states predicted by the model to experience welfare losses tend to be states where the Republican candidate performed worse, and vice versa.

There is also a positive relationship between the Republican vote share in 2016 and the model-predicted welfare change, as shown in Figure 6 and Table 3. The regression results indicate that states with larger predicted welfare gains tend to exhibit higher Republican vote shares.

These patterns may help researchers better understand the rationale underlying individuals' voting decisions.

6 Conclusion

This paper shows that allowing one production factor—capital—to be mobile across regions fundamentally alters the welfare implications of trade liberalization, particularly for wage earners. In the absence of factor mobility, trade liberalization leads to welfare gains for all agents, as demonstrated by Arkolakis et al. (2012), since domestic production patterns

[8]The validation for the model predicted number of firms can be found in Appendix J

remain unchanged. However, when capital is mobile across regions, trade liberalization induces competition for capital across locations. This competition affects the average cost of establishing new firms, which in turn influences domestic production.

An increase in tariffs, particularly when implemented unilaterally, can provide a competitive advantage in attracting capital. This advantage lowers the cost of establishing new firms domestically, leading to greater firm entry and higher domestic production. When domestic production responds to trade policy, welfare gains from trade are no longer guaranteed.

However, the model is purely static and therefore does not incorporate several potentially important economic mechanisms, such as asset markets, price rigidities, or financial market frictions. Accounting for these elements could alter the welfare effects discussed above.

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Tables

Table 2: Tariff welfare validation: Model vs. Melitz (HC3 robust SEs)

	(1) %Δ worker welfare (my model)	(2) %Δ worker welfare (Melitz)	(3) %Δ worker welfare (my model)	(4) %Δ worker welfare (Melitz)	(5) %Δ RPI
%Δ RPCE	-38.012***	-14.773			1.029***
	(11.515)	(14.187)			(0.060)
%Δ RPCE (2016–2017)			8.925	-17.096	
			(12.890)	(15.553)	
%Δ RPI	30.566***	15.241			
	(10.262)	(14.954)			
%Δ RPI (2016–2017)			-12.804	5.312	
			(9.132)	(4.801)	
%Δ worker welfare (model)					0.005**
					(0.002)
%Δ worker welfare (Melitz)					0.003
					(0.006)
Constant	1.610***	-0.225	1.627***	0.108	-0.005
	(0.165)	(0.153)	(0.273)	(0.198)	(0.005)
Observations	51.000	51.000	51.000	51.000	51.000
R-squared	0.239	0.087	0.035	0.082	0.875
Adj. R-squared	0.207	0.049	-0.006	0.044	0.867

Table 3: Republican vote share vs. model-predicted welfare change

	(1) Republican Vote Share
Δ worker welfare (model)	0.164**
	(0.074)
Constant	0.491***
	(0.015)
Observations	51.000
R-squared	0.234
Adj. R-squared	0.218

Figures

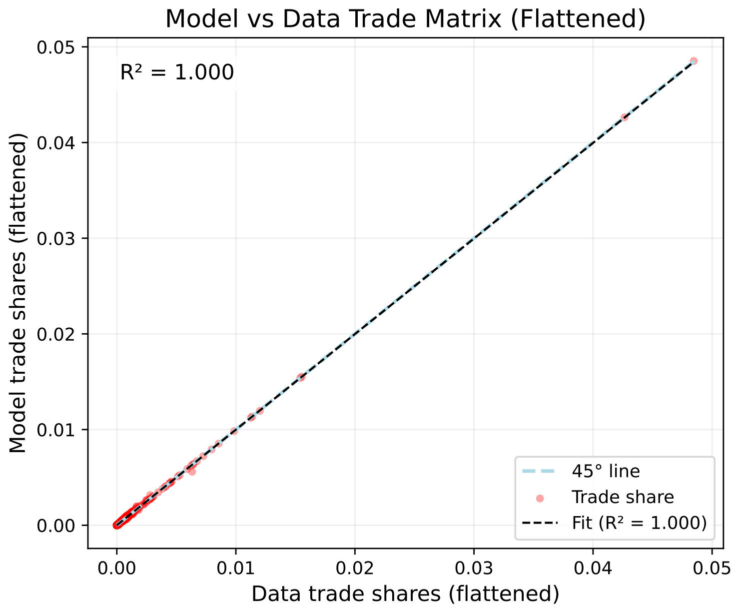

Figure 1: Model-implied versus data trade-share moments (2017).

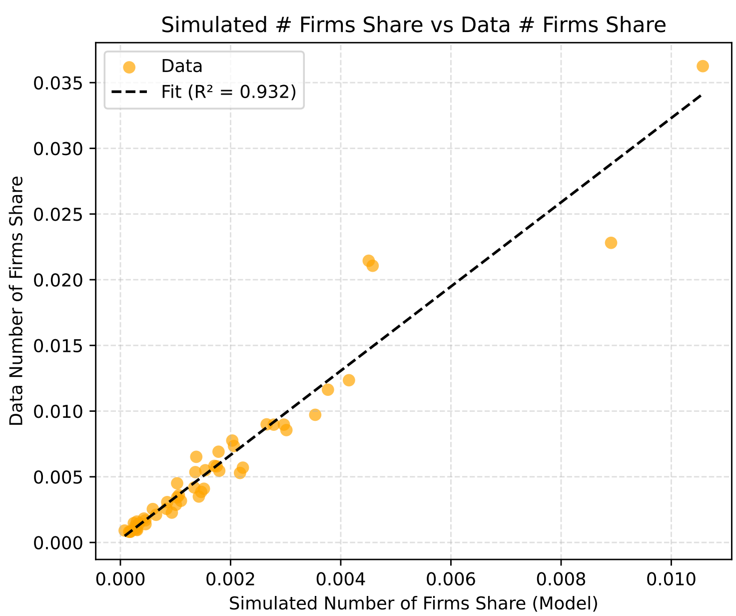

Figure 2: Model-implied versus data number of establishments share moments (2017).

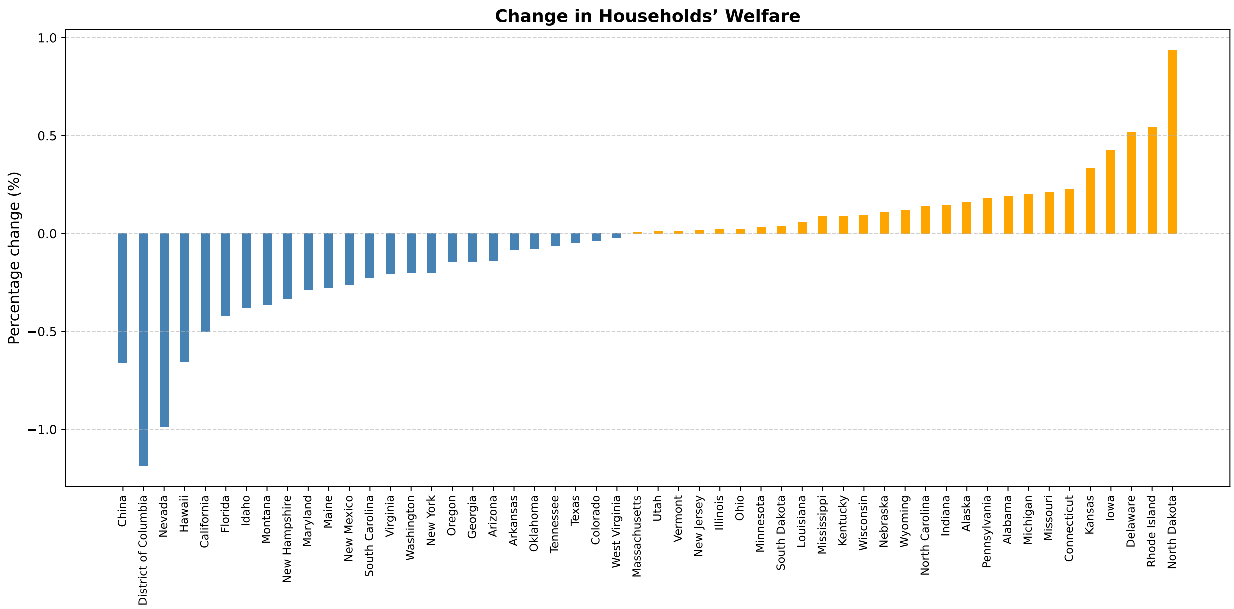

Figure 3: Model-predicted household welfare changes under the 2018 U.S. tariff policy.

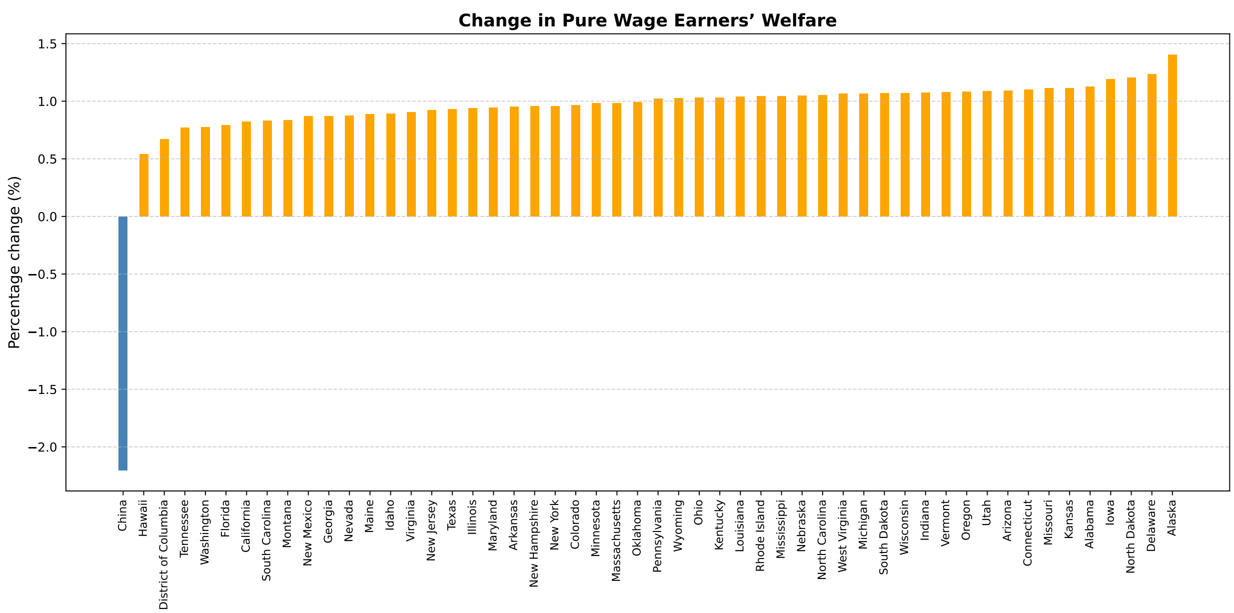

Figure 4: Model-predicted household welfare changes under the 2018 U.S. tariff policy.

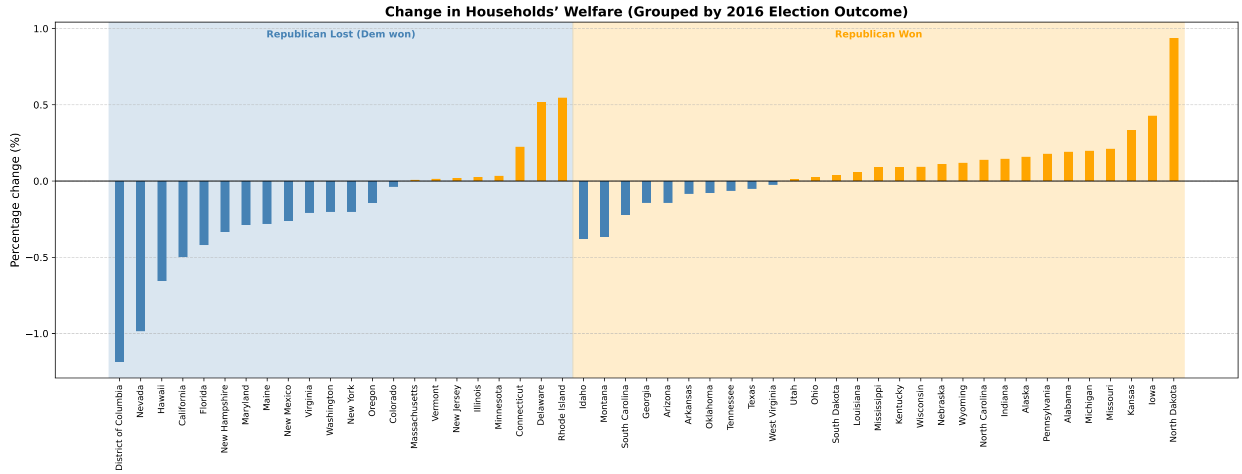

Figure 5: Model-predicted household welfare changes vs. Republican victory in the 2016 election.

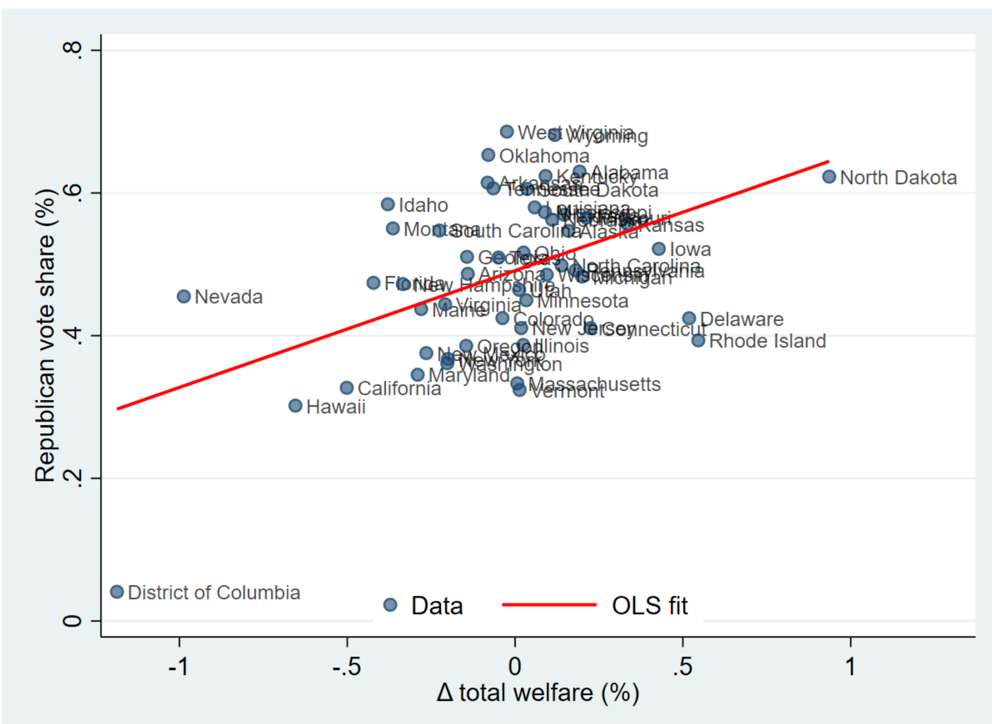

Figure 6: Relationship between Republican vote share in 2016 and model-predicted welfare changes.

A Proof for symmetric equilibrium solution

Under the totally symmetric assumption, I have

$$\gamma = \frac{1}{2}$$

$$F_J^E = F_L^E = F^E$$

$$F_{JJ} = F_{LL} = F_0$$

$$F_{JL} = F_{LJ} = F_1$$

$$L_J = L_L = L$$

$$\tau_{JL} = \tau_{LJ} = \tau$$

$$\tau_{JJ} = \tau_{LL} = 0$$

Combining the labor market clearing conditions and the free entry conditions gives the expression for the number of firms as a function of wages:

$$N_S = \frac{\sigma - 1}{\sigma\alpha} \frac{W_S^{1-\xi} L}{\xi F^E}. \tag{48}$$

Substituting Equation 48 into the expression for aggregated trade values, Equation 4, gives:

$$V_{SI} = \frac{(\sigma - 1)\underline{z}^{\alpha}}{\alpha + 1 - \sigma} \frac{L}{\xi F^E} W_S^{1-\xi\epsilon} B_I^{\frac{\alpha}{\sigma-1}} F_{SI}^{1+\frac{\alpha}{1-\sigma}} (1 + \tau_{SI})^{-\alpha}. \tag{49}$$

Recall that I need to solve for $\{W_J, W_L, B_J, B_L\}$ using two labor market clearing conditions (one for each country), one global capital market clearing condition, and one country's trade balance condition, which are:

$$\sum_I V_{JI} = \frac{W_J L}{\xi},$$

$$\sum_I V_{LI} = \frac{W_L L}{\xi},$$

$$rK_0 = (1 - \xi)(\sum_I V_{JI} + \sum_I V_{LI}),$$

$$W_J L + \gamma r K_0 = V_{JJ} + V_{LJ}.$$

The proportional relationship between V_{JJ} and V_{JL} is:

$$\frac{V_{JJ}}{V_{JL}} = \frac{B_J^{\frac{\alpha}{\sigma-1}} F_{JJ}^{1+\frac{\alpha}{1-\sigma}}}{B_L^{\frac{\alpha}{\sigma-1}} F_{JL}^{1+\frac{\alpha}{1-\sigma}} (1+\tau_{JL})^{-\alpha}} = \left(\frac{B_J}{B_L}\right)^{\frac{\alpha}{\sigma-1}} C,$$

where

$$C = \left(\frac{F_0}{F_1}\right)^{1+\frac{\alpha}{1-\sigma}} (1+\tau)^{\alpha}.$$

Similarly,

$$\frac{V_{LL}}{V_{LJ}} = \left(\frac{B_L}{B_J}\right)^{\frac{\alpha}{\sigma-1}} C.$$

Denote

$$\beta = \left(\frac{B_J}{B_L}\right)^{\frac{\alpha}{\sigma-1}}.$$

Then I have

$$V_{JJ} = \beta C V_{JL}, \tag{50}$$

$$V_{LL} = \frac{C}{\beta} V_{LJ}. \tag{51}$$

At the same time,

$$\frac{V_{JL}}{V_{LJ}} = \frac{W_J^{1-\xi\epsilon}}{W_L^{1-\xi\epsilon}} \frac{B_L^{\frac{\alpha}{\sigma-1}} F_{JL}^{1+\frac{\alpha}{1-\sigma}} (1+\tau_{JL})^{-\alpha}}{B_J^{\frac{\alpha}{\sigma-1}} F_{LJ}^{1+\frac{\alpha}{1-\sigma}} (1+\tau_{LJ})^{-\alpha}} = \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon} \left(\frac{B_L}{B_J}\right)^{\frac{\alpha}{\sigma-1}}.$$

So I have

$$\frac{V_{JL}}{V_{LJ}} = \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon} \frac{1}{\beta}. \tag{52}$$

Combining the two labor market clearing conditions gives:

$$\frac{W_J}{W_L} = \frac{V_{JJ} + V_{JL}}{V_{LL} + V_{LJ}} = \frac{V_{JL}(1 + C\beta)}{V_{LJ}(1 + \frac{C}{\beta})}. \tag{53}$$

Substituting Equation 52 into Equation 53 gives:

$$\frac{W_J}{W_L} = \frac{1}{\beta} \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon} \frac{1 + C\beta}{1 + \frac{C}{\beta}}. \tag{54}$$

This further implies

$$\left(\frac{W_J}{W_L}\right)^{\xi\epsilon} = \frac{1 + C\beta}{\beta + C}. \tag{55}$$

Now combining Equation 50 and Equation 51 gives:

$$\frac{V_{JJ}}{V_{LL}} = \frac{\beta C V_{JL}}{\frac{C}{\beta} V_{LJ}} = \beta^2 \frac{V_{JL}}{V_{LJ}} = \beta \left(\frac{W_J}{W_L} \right)^{1-\xi\epsilon}. \tag{56}$$

Substituting Equation 55 back gives:

$$\frac{V_{JJ}}{V_{JL}} = \frac{W_J}{W_L} \frac{\beta^2 + C\beta}{1 + C\beta}, \tag{57}$$

and

$$\frac{V_{JL}}{V_{LJ}} = \frac{W_J}{W_L} \frac{\beta + C}{\beta + C\beta^2}. \tag{58}$$

Recall that combining the labor market clearing conditions gives:

$$\frac{W_J}{W_L} = \frac{V_{JJ} + V_{JL}}{V_{LL} + V_{LJ}}.$$

Representing V_{JJ} using V_{LL} and V_{JL} using V_{LJ} gives:

$$\frac{W_J}{W_L} = \frac{\frac{W_J}{W_L} \frac{\beta^2 + C\beta}{1 + C\beta} V_{LL} + \frac{W_J}{W_L} \frac{\beta + C}{\beta + C\beta^2} V_{LJ}}{V_{LL} + V_{LJ}}.$$

This further implies

$$1 = \frac{\frac{\beta^2 + C\beta}{1 + C\beta} V_{LL} + \frac{\beta + C}{\beta + C\beta^2} V_{LJ}}{V_{LL} + V_{LJ}}. \tag{59}$$

For this equation to hold, it must be that

$$\frac{\beta^2 + C\beta}{1 + C\beta} = \frac{\beta + C}{\beta + C\beta^2} = 1, \tag{60}$$

which implies $\beta = 1$, i.e. $B_J = B_L$. From Equation 55, this also implies $W_J = W_L$.

Using $B_J = B_L = B$ and $W_J = W_L = W$, I can solve for

$$W = \frac{L\xi}{2K_0(1-\xi)}$$

and

$$B^{\frac{\alpha}{\sigma-1}} = \left(\frac{L\xi}{2K_0(1-\xi)} \right)^{\xi\epsilon} \frac{\alpha + 1 - \sigma}{(\sigma - 1)\underline{z}^{\alpha}} \frac{F^E}{F_0^{1+\frac{\alpha}{1-\sigma}} + F_1^{1+\frac{\alpha}{1-\sigma}} (1+\tau)^{-\alpha}}.$$

Therefore, the solution of the model under symmetric equilibrium is as follows:

$$\begin{pmatrix} W_J \\ W_L \\ B_J \\ B_L \end{pmatrix} = \begin{pmatrix} \frac{L\xi}{2K_0(1-\xi)} \\ \frac{L\xi}{2K_0(1-\xi)} \\ \left[\left(\frac{L\xi}{2K_0(1-\xi)}\right)^{\xi\epsilon} \frac{\alpha+1-\sigma}{(\sigma-1)\underline{z}^{\alpha}} \frac{F^E}{F_0^{1+\frac{\alpha}{1-\sigma}}+F_1^{1+\frac{\alpha}{1-\sigma}}(1+\tau)^{-\alpha}}\right]^{\frac{\sigma-1}{\alpha}} \\ \left[\left(\frac{L\xi}{2K_0(1-\xi)}\right)^{\xi\epsilon} \frac{\alpha+1-\sigma}{(\sigma-1)\underline{z}^{\alpha}} \frac{F^E}{F_0^{1+\frac{\alpha}{1-\sigma}}+F_1^{1+\frac{\alpha}{1-\sigma}}(1+\tau)^{-\alpha}}\right]^{\frac{\sigma-1}{\alpha}} \end{pmatrix}.$$

B Immobile Capital

This appendix considers a variant of the model in Section 2 in which capital is immobile across countries. I show that, under immobile capital, the equilibrium mass of firms is pinned down by exogenous parameters rather than adjusting to endogenous equilibrium variables.

As in Section 2, the total world capital endowment is K_0. Households in country J own a share γ of this endowment, while households in country L own the remaining share $1-\gamma$. The parameters K_0, γ, and $1-\gamma$ are all exogenously given.

B.1 Consumer Side

Consumers (households) in destination country $I \in \{J, L\}$ solve the same utility-maximization problem subject to their budget constraint as in Section 2, except that the rental rate of capital is now country specific. In particular, instead of a common rental rate r, consumers in country I face r_I.

Given this modification, the demand for variety ω by consumers in country I is

$$q_I(\omega) = \frac{p_I(\omega)^{-\sigma}}{P_I^{1-\sigma}} \left(r_I K_I + W_I L_I\right).$$

B.2 Firm Side

On the firm side, the only modification relative to Section 2 is that the rental rate of capital is country specific. Firms located in country $S \in \{J, L\}$ therefore face the rental rate r_S.

As illustrated in Section 2, with constant source–destination iceberg trade costs, there is a one-to-one mapping between the quantity consumed in destination I, $q_I(\omega)$, and the quantity shipped by a firm with productivity z from source S to destination I, $q_{SI}(z)$.

Given demand for goods shipped to destination I, a firm located in country S requires the following inputs:

$$K_{SI}(z) = (1-\xi)\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r_S}\right)^{\xi}(1+\tau_{SI}),$$

$$L_{SI}(z) = \xi\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r_S}\right)^{\xi-1}(1+\tau_{SI}).$$

Finally, under monopolistic competition, firms set prices as a constant markup over marginal cost:

$$p_{SI}(z) = \frac{\sigma}{\sigma-1}\frac{W_S^{\xi}r_S^{1-\xi}}{z}(1+\tau_{SI}).$$

B.3 Aggregated Trade Values

The trade value generated by a firm with productivity z located in country S and selling to destination I is

$$V_{SI}(z) = \sigma B_I\left(\frac{W_S^{\xi}r_S^{1-\xi}}{z}(1+\tau_{SI})\right)^{1-\sigma}.$$

Aggregating over all firms in country S that export to market I, total trade values are given by

$$V_{SI} = \frac{\sigma\alpha\underline{z}^{\alpha}}{\alpha+1-\sigma}N_S\,B_I^{\frac{\alpha}{\sigma-1}}\,F_{SI}^{1+\frac{\alpha}{1-\sigma}}\left(W_S^{\xi}r_S^{1-\xi}\right)^{1-\epsilon}(1+\tau_{SI})^{-\alpha}. \tag{61}$$

Relative to Equation 4, the only difference is that the rental rate of capital is now country specific.

B.4 Technology Threshold for Selling in a Market

The productivity threshold for a firm located in country S to profitably sell in destination market I is analogous to Equation 3, except that the rental rate of capital is now country specific. Specifically,

$$z_{\min}^{SI} = \left(\frac{F_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}}\left(W_S^{\xi}r_S^{1-\xi}\right)^{\frac{\sigma}{\sigma-1}}(1+\tau_{SI}).$$

B.5 Equilibrium Conditions

With immobile capital, the global capital-market–clearing condition is replaced by country-specific capital-market–clearing conditions, each associated with its own rental rate. Normalizing the wage in country J to one, the endogenous variables are

$$W_L,\ r_J,\ r_L,\ N_J,\ N_L,\ B_J,\ B_L.$$

These seven endogenous variables are pinned down by seven equilibrium conditions. For each country, these include:

1. a free-entry condition,

2. a labor-market–clearing condition, and

3. a capital-market–clearing condition.

In addition, one trade-balance condition closes the model.

The free-entry conditions are

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{JI} = N_J F_J^E W_J^{\xi} r_J^{1-\xi}, \tag{62}$$

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{LI} = N_L F_L^E W_L^{\xi} r_L^{1-\xi}. \tag{63}$$

Labor-market–clearing conditions are

$$W_J L_J = \xi \sum_I V_{JI}, \tag{64}$$

$$W_L L_L = \xi \sum_I V_{LI}. \tag{65}$$

Capital-market–clearing conditions are

$$r_J K_J = r_J \gamma K_0 = (1-\xi)\sum_I V_{JI}, \tag{66}$$

$$r_L K_L = r_L (1-\gamma) K_0 = (1-\xi)\sum_I V_{LI}. \tag{67}$$

Finally, the trade-balance condition is

$$V_{JL} = V_{LJ}. \tag{68}$$

Notice that combining the country-specific labor-market–clearing condition with the capital-market–clearing condition implies

$$\frac{W_S}{r_S} = \frac{\xi}{1-\xi}\frac{K_S}{L_S}. \tag{69}$$

Equation (69) shows that the relative factor price is pinned down by factor endowments, which are exogenous. As a result, the rental rate r_S can be expressed as a function of the wage W_S, allowing r_S to be eliminated as an independent equilibrium object. The set of endogenous variables can therefore be reduced to

$$\{W_L,\, N_J,\, N_L,\, B_J,\, B_L\}.$$

Next, combining the country-specific free-entry condition with the labor-market–clearing condition yields

$$N_S = \frac{\sigma-1}{\sigma\alpha}\frac{L_S}{\xi F_S^E}\left(\frac{W_S}{r_S}\right)^{1-\xi}. \tag{70}$$

Substituting Equation (69) into Equation (70) gives

$$N_S = \frac{\sigma-1}{\sigma\alpha}\frac{1}{\xi F_S^E}\left(\frac{\xi}{1-\xi}\right)^{1-\xi} K_S^{1-\xi}L_S^{\xi}. \tag{71}$$

Equation (71) implies that the equilibrium mass of firms N_S depends solely on exogenous factor endowments and structural parameters. Consequently, N_J and N_L can also be eliminated as equilibrium objects, further reducing the set of endogenous variables to

$$\{W_L,\, B_J,\, B_L\}.$$

Equivalently, after substituting out r_S and N_S, the equilibrium system is isomorphic to one in which labor is the only immobile input, with capital affecting equilibrium outcomes only through the composite term $K_S^{1-\xi}L_S^{\xi}$.

As a result, to determine the remaining endogenous variables $\{W_L, B_J, B_L\}$, it suffices to impose the two country-specific labor-market–clearing conditions together with the trade-balance condition:

$$W_J L_J = \xi \sum_I V_{JI}, \tag{72}$$

$$W_L L_L = \xi \sum_I V_{LI}, \tag{73}$$

$$V_{JL} = V_{LJ}. \tag{74}$$

Number of firms From Equation (70), all variables on the right-hand side are exogenous. In particular, K_S and L_S are fixed country-specific factor endowments, while ξ and F_S^E are structural parameters. Therefore, under immobile capital, the equilibrium mass of firms in country $S \in \{J, L\}$ is pinned down entirely by exogenous primitives and does not respond to endogenous equilibrium outcomes such as wages, rental rates, or market size. This result highlights that it is the interaction between capital usage in production and capital mobility that causes firm entry to respond endogenously to factor prices, as shown in Section 2.

B.6 Comparative Statics

Log-differentiate Equation 61 gives:

$$\hat{V}_{SI} = \frac{\alpha}{\sigma - 1}\hat{B}_I + (1-\epsilon)\xi\hat{W}_S + (1-\xi)(1-\epsilon)\hat{r}_S - \alpha\hat{\tau}_{SI}$$

Since $\frac{W_S}{r_S}$ is equal to a constant, I have

$$\hat{W}_S = \hat{r}_S$$

So

$$\hat{V}_{SI} = \frac{\alpha}{\sigma - 1}\hat{B}_I + (1-\epsilon)\hat{W}_S - \alpha\hat{\tau}_{SI} \tag{75}$$

Given this, I can then log-differentiate equilibrium conditions: For country J's labor market:

$$\sum_I V_{SI}\hat{W}_S = \sum_I V_{SI}\hat{V}_{SI}$$

Denote $\lambda_S = \frac{V_{SI}}{\sum_I V_{SI}}$, under symmetry, $\lambda_J = \lambda_L = \lambda$ So I have:

$$\hat{W}_J = \lambda\hat{V}_{JL} + (1-\lambda)\hat{V}_{JJ} \tag{76}$$

Sicne W_J is the numeraire, this furtherly becomes:

$$\lambda\hat{V}_{JL} = (\lambda - 1)\hat{V}_{JJ} \tag{77}$$

For country L:

$$\hat{W}_L = \lambda\hat{V}_{LJ} + (1-\lambda)\hat{V}_{LL} \tag{78}$$

Finally for the trade balance condition

$$\hat{V}_{JL} = \hat{V}_{LJ} \tag{79}$$

Combining them together, the system can be written in matrix form as

$$\begin{pmatrix} 0 & -\dfrac{\alpha(1-\lambda)}{\sigma-1} & -\dfrac{\alpha\lambda}{\sigma-1} \\ \epsilon & -\dfrac{\alpha\lambda}{\sigma-1} & -\dfrac{\alpha(1-\lambda)}{\sigma-1} \\ 1-\epsilon & \dfrac{\alpha}{\sigma-1} & \dfrac{\alpha}{\sigma-1} \end{pmatrix} \begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} -\alpha\lambda\,\hat{\tau}_{JL} \\ -\alpha\lambda\,\hat{\tau}_{LJ} \\ \alpha(\hat{\tau}_{LJ}-\hat{\tau}_{JL}) \end{pmatrix}.$$

Bilateral trade liberalization Suppose trade costs fall symmetrically,

$$\hat{\tau}_{JL} = \hat{\tau}_{LJ} = \hat{\tau} < 0.$$

Solving the system yields

$$\begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda(\sigma-1)\hat{\tau} \\ \lambda(\sigma-1)\hat{\tau} \end{pmatrix}.$$

Using the relationship $-\hat{P}_I = -\hat{B}_I/(\sigma-1)$, both countries experience identical welfare gains equal to

$$-\lambda\hat{\tau}.$$

Hence, the welfare gains from bilateral trade liberalization coincide with those obtained in Section 2.

Unilateral trade liberalization Now consider a unilateral reduction in trade costs from country L to country J,

$$\hat{\tau}_{LJ} = \hat{\tau} < 0, \qquad \hat{\tau}_{JL} = 0.$$

The solution to the system is

$$\begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} -\dfrac{\alpha(1-\lambda)\hat{\tau}}{A} \\ (\sigma-1)\dfrac{\lambda B\,\hat{\tau}}{A} \\ -(\sigma-1)\dfrac{(1-\lambda)B\,\hat{\tau}}{A} \end{pmatrix},$$

where $A \equiv (\epsilon-1)(1-2\lambda)$ and $B \equiv \epsilon(1-\lambda)+\lambda$.

From Equation (76), it is immediate that in this environment trade-cost changes do not affect domestic GDP. In contrast, as shown in Section 2, Paragraph 2.9.4, a unilateral tariff change does alter domestic GDP when capital is mobile. This difference highlights the role of capital mobility in transmitting trade-policy shocks to aggregate production.

The domestic price index change will be simply

$$\hat{P}_J = \frac{1}{\sigma - 1}\hat{B}_J \tag{80}$$

where substituting the solution gives

$$\hat{P}_J = -\frac{\lambda}{1-\lambda}\hat{B}_L \tag{81}$$

C Different Input Factor Intensity for Fixed Costs

Suppose that the factor intensity of fixed costs differs from that of variable production. Specifically, firms produce final goods using the Cobb–Douglas technology

$$z\left(\frac{l}{\xi}\right)^{\xi}\left(\frac{k}{1-\xi}\right)^{1-\xi},$$

where $\xi \in (0,1)$ denotes the labor share in variable production.

In contrast, fixed costs are incurred using a different factor intensity. To serve destination I, firms located in country S must incur a fixed cost that uses labor and capital according to

$$\frac{1}{F_{SI}}\left(\frac{l}{\phi}\right)^{\phi}\left(\frac{k}{1-\phi}\right)^{1-\phi},$$

where $\phi \in (0,1)$ denotes the labor share in fixed costs. As a result, the fixed cost of serving market I for firms located in country S is given by

$$F_{SI}W_S^{\phi}r^{1-\phi}.$$

Similarly, the fixed cost of entry for firms located in country S is given by

$$F_S^E W_S^{\phi}r^{1-\phi}.$$

C.1 Consumer Side, Firm Side, and Trade Values

Allowing fixed costs to have a different factor intensity does not affect variable-cost–related calculations. As a result, consumer demand, firm pricing, and factor demands associated with variable production remain unchanged.

In particular, the quantity demanded of a variety with productivity z shipped from country S to country I is

$$q_{SI}(z) = \frac{\sigma}{\sigma-1}\frac{W_S^{\xi}r^{1-\xi}}{z}(1+\tau_{SI}).$$

The firm's factor demands for variable production are therefore unchanged and given by

$$K_{SI}(z) = (1-\xi)\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r}\right)^{\xi}(1+\tau_{SI}),$$

$$L_{SI}(z) = \xi\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r}\right)^{\xi-1}(1+\tau_{SI}).$$

The optimal price charged by a firm from country S selling to country I is

$$p_{SI}(z) = \frac{\sigma}{\sigma-1}\frac{W_S^{\xi}r^{1-\xi}}{z}(1+\tau_{SI}).$$

Finally, revenue earned by a firm with productivity draw z selling from S to I is given by

$$V_{SI}(z) = \sigma B_I\left(W_S^{\xi}r^{1-\xi}(1+\tau_{SI})\right)^{1-\sigma}z^{\sigma-1}.$$

All expressions above are identical to those in Section 2.

C.2 Technology Threshold and Aggregate Trade Value

The first deviation from Section 2 concerns the productivity threshold for firms located in country S to serve market I. With a different factor intensity for fixed costs, the zero-profit cutoff condition is now given by

$$\tilde{\pi}_{SI}\left(z_{SI}^{\min}\right) = F_{SI}W_S^{\phi}r^{1-\phi}.$$

This condition implies the following productivity threshold:

$$z_{SI}^{\min} = \left(\frac{F_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}}W_S^{\frac{\phi}{\sigma-1}+\xi}r^{\frac{1-\phi}{\sigma-1}+1-\xi}(1+\tau_{SI}).$$

Aggregate trade flows from country S to market I are therefore given by

$$V_{SI} = \frac{\sigma\alpha_S\underline{z}_S^{\alpha_S}}{\alpha_S+1-\sigma}N_S\,F_{SI}^{1+\frac{\alpha_S}{1-\sigma}}\,B_I^{\frac{\alpha_S}{\sigma-1}}\,(1+\tau_{SI})^{-\alpha_S}\,W_S^{\phi-\frac{\alpha_S\phi}{\sigma-1}-\alpha_S\xi}\,r^{1-\phi-\frac{\alpha_S(1-\phi)}{\sigma-1}-\alpha_S(1-\xi)}.$$

C.3 CRS Properties for Factor Prices and Market-Clearing Conditions

We first verify that the cost function remains homogeneous of degree one in factor prices when fixed costs have a different factor intensity. Using Equation **??** as the general form, the total cost incurred by a firm from country S selling to market I can be written as

$$C(q, W_S, r) = F_{SI} W_S^{\phi} r^{1-\phi} + F_S^E W_S^{\phi} r^{1-\phi} + W_S^{\xi} r^{1-\xi} \frac{q_{SI}(z)(1+\tau_{SI})}{z}.$$

It follows immediately that

$$C(q, \lambda W_S, \lambda r) = \lambda C(q, W_S, r),$$

so the cost function remains homogeneous of degree one in factor prices (W_S, r). Equilibrium allocations therefore depend only on relative factor prices.

Turning to market-clearing conditions, labor is used in variable production as well as in the payment of fixed costs for market access and entry. Under the symmetric-country assumption, total labor used by firms from country S to serve destination I can be written as

$$\tilde{L}_{SI} = \frac{\xi(\sigma-1)}{\sigma W_S} V_{SI} + \frac{\phi(\alpha+1-\sigma)}{\sigma\alpha W_S} V_{SI}.$$

Aggregating across destinations and adding entry fixed costs yields total labor demand in country S:

$$L_S = \left[\frac{(\sigma-1)\xi+\phi}{\sigma W_S}\right] \sum_I V_{SI}.$$

Hence, labor-market–clearing conditions can be written compactly as

$$L_J = \frac{(\sigma-1)\xi+\phi}{\sigma W_J} \sum_I V_{JI}, \qquad L_L = \frac{(\sigma-1)\xi+\phi}{\sigma W_L} \sum_I V_{LI}.$$

Similarly, capital is used in variable production and in the payment of fixed costs. The global capital-market–clearing condition is therefore given by

$$K_0 = \frac{(\sigma-1)(1-\xi)+(1-\phi)}{\sigma r} \sum_S \sum_I V_{SI}.$$

Note that when $\phi = \xi$, the market-clearing conditions collapse to those in Section 2.

C.4 Firm Entry Decisions

Combining the free-entry condition

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{SI} = N_S F_S^E W_S^{\phi} r^{1-\phi}$$

with the labor-market–clearing condition

$$\sigma W_S L_S = ((\sigma-1)\xi+\phi)\sum_I V_{SI}$$

yields the equilibrium mass of firms:

$$N_S = \frac{(\sigma-1)L_S}{\alpha F_S^E\left((\sigma-1)\xi+\phi\right)}\left(\frac{W_S}{r}\right)^{1-\phi}.$$

As in the baseline model, firm entry depends on relative factor prices. A higher rental–wage ratio raises the cost of entry thus reducing the equilibrium mass of firms. Allowing fixed costs to have a different factor intensity alters the elasticity of entry with respect to factor prices but does not change the underlying mechanism.

D Constant Fixed Costs

This appendix considers the case in which the fixed cost of serving market I from country S, denoted f_{SI}, and the fixed cost of operating (entry), denoted f_S^E, are both constants (i.e., they do not scale with factor prices). Since the rental rate of capital is chosen as the numeraire, these fixed costs are denominated in units of capital and therefore do not vary with equilibrium wages. Equivalently, this specification corresponds to setting the wage intensity parameter ϕ introduced in Appendix C equal to zero. This modification does not affect the household problem or firms' pricing rules, but it alters the export cutoff and the free-entry condition.

Revenue at the firm level. For a firm in S with productivity z selling to market I, revenue is unchanged:

$$V_{SI}(z) = \sigma B_I\Big(W_S^{\xi} r^{1-\xi}(1+\tau_{SI})\Big)^{1-\sigma} z^{\sigma-1}.$$

Market-access cutoff. Let $\pi_{SI}(z)$ denote operating profits (before paying the market-specific fixed cost). With CES demand and monopolistic competition,

$$\pi_{SI}(z) = \frac{1}{\sigma} V_{SI}(z).$$

The cutoff productivity $z_{\min}^{SI}$ is pinned down by the zero-profit condition

$$\pi_{SI}(z_{\min}^{SI}) = f_{SI},$$

which implies

$$z_{\min}^{SI} = \left(\frac{\sigma f_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}} W_S^{\xi} r^{1-\xi}(1+\tau_{SI}).$$

Aggregate trade value. The aggregate value of sales from S to I is

$$V_{SI} = N_S \int_{z_{\min}^{SI}}^{\infty} V_{SI}(z) f^S(z)\, dz = \frac{\sigma \alpha_S}{\alpha_S + 1 - \sigma} N_S\, \underline{z}_S^{\alpha_S}\, B_I \Big(W_S^{\xi} r^{1-\xi}(1+\tau_{SI})\Big)^{1-\sigma} \left(z_{\min}^{SI}\right)^{\sigma-1-\alpha_S}.$$

Substituting the cutoff yields

$$V_{SI} = N_S \frac{\sigma \alpha_S \underline{z}^{\alpha_S}}{\alpha_S + 1 - \sigma} f_{SI}^{1+\frac{\alpha_S}{1-\sigma}} B_I^{\frac{\alpha_S}{\sigma-1}} (\Big(W_S^{\xi} r^{1-\xi}(1+\tau_{SI})\Big)^{-\alpha_S}$$

Profits net of market-access fixed costs. Profits net of the market-specific fixed cost are

$$\tilde{\pi}_{SI}(z) = \pi_{SI}(z) - f_{SI} = \frac{V_{SI}(z)}{\sigma} - f_{SI}.$$

Aggregating across active firms,

$$\tilde{\pi}_{SI} = N_S \int_{z_{\min}^{SI}}^{\infty} \tilde{\pi}_{SI}(z) f^S(z)\, dz = \frac{\sigma - 1}{\sigma \alpha_S} V_{SI},$$

where the last equality uses the Pareto assumption and the cutoff defined by $\pi_{SI}(z_{\min}^{SI}) = f_{SI}$.

Free-entry condition. Let total expected operating profits net of market-specific fixed costs be $\sum_I \tilde{\pi}_{SI}$. With constant entry cost f_S^E, the free-entry condition is

$$\sum_I \tilde{\pi}_{SI} = N_S f_S^E, \qquad \text{equivalently} \qquad \sum_I \frac{\sigma-1}{\sigma \alpha_S} V_{SI} = N_S f_S^E.$$

Labor market clearing. Labor is used only in variable production. Variable labor demand for a firm with productivity z serving I is proportional to revenue:

$$W_S L_{SI}(z) = \xi \frac{\sigma - 1}{\sigma} V_{SI}(z).$$

Aggregating across destinations and active firms yields

$$W_S L_S = \sum_I N_S \int_{z_{\min}^{SI}}^{\infty} W_S L_{SI}(z) f^S(z)\, dz = \xi \frac{\sigma - 1}{\sigma} \sum_I V_{SI}.$$

Combining labor market clearing with the free-entry condition gives an expression for the equilibrium mass of entrants:

$$N_S = \frac{\sigma - 1}{\sigma \alpha_S} \frac{\sum_I V_{SI}}{f_S^E} = \frac{W_S L_S}{\xi \alpha_S f_S^E}.$$

Takeaway. This appendix highlights a general implication of combining heterogeneous-firm free entry with an integrated capital market. Regardless of whether fixed costs are specified in units of factors (e.g., $W_S^{\xi} r^{1-\xi}$) or as constants, the equilibrium mass of entrants N_S is no longer pinned down solely by endowments and parameters.

E Analysis of the elasticity

In an Armington-style model, regardless of capital mobility, the elasticity is

$$1 - \sigma < 0.$$

Chaney (2008) shows that within the Melitz (2003) framework, assuming a Pareto distribution of firm productivity and no firm entry or exit, the elasticity takes the form

$$-\alpha < 0.$$

If firm entry and exit are allowed within the Chaney (2008) framework, the elasticity becomes

$$1 - \epsilon < 0.$$

How the difference arises To clarify the source of these differences, note first that the price index is a function of domestic and foreign unit factor prices. With the rental rate r chosen as the numeraire, unit factor prices depend solely on country-specific wages. Consequently, all equilibrium adjustments are ultimately driven by wage changes.

However, the log-differential of the price index is not linear in the log-differentials of wages. I therefore separate the effect of wage changes from the effect operating through the price index. When discussing the influence of wages on trade values, I refer to the *direct* effect of wages holding the price index fixed, as opposed to the *indirect* effects transmitted through changes in the price index (or, equivalently, B_I).

To build intuition, I begin at the firm level. Consider a firm whose productivity z is sufficiently high that its market participation decisions are unaffected by a potential tariff change. Log-differentiating Equation 1 yields

$$\hat{V}_{SI}(z) = \xi(1-\sigma)\hat{W}_S + (1-\xi)(1-\sigma)\hat{r} + \hat{B}_I + (1-\sigma)\hat{\tau}_{SI}. \tag{82}$$

When capital is immobile, the rental rate is country-specific. As shown in Appendix B, relative factor prices are pinned down by fixed factor endowments, implying $\hat{r}_S = \hat{W}_S$. Substituting this condition yields

$$\hat{V}_{SI}(z) = (1-\sigma)\hat{W}_S + \hat{B}_I + (1-\sigma)\hat{\tau}_{SI}. \tag{83}$$

Holding wages fixed, firm-level trade values admit a partial-equilibrium interpretation. Demand in market I is downward sloping, while supply is perfectly elastic at the constant-markup price implied by marginal cost. Firm-level sales are therefore determined by the intersection of a downward-sloping demand curve and a horizontal supply curve. Trade values reflect the resulting trade-off between prices and quantities as wages change.

In this case, the elasticity of firm-level sales with respect to wages is $1-\sigma$, exactly as in the Armington (1969) model. In the absence of heterogeneous-firm entry and exit, this firm can be interpreted as a representative firm for the country, so its sales elasticity coincides with the aggregate elasticity, replicating the Armington result.

If we go one step further and allow for heterogeneous firms, the relevant object becomes aggregate trade values. Log-differentiating Equation 4 yields

$$\hat{V}_{SI} = \hat{N}_S + \hat{B}_I + \xi(1-\sigma)\hat{W}_S + (1-\xi)(1-\sigma)\hat{r} + (1-\sigma)\hat{\tau}_{SI} + (\sigma-\alpha-1)\hat{Z}^{SI}_{\min}. \tag{84}$$

Under the assumption that the fixed cost is a constant instead of responding to the factor price, as shown in Chaney (2008), this implies

$$\hat{Z}^{SI}_{\min} = \xi\hat{W}_S + (1-\xi)\hat{r} + \frac{1}{1-\sigma}\hat{B}_I + \hat{\tau}_{SI}. \tag{85}$$

Factors are immobile so that $\hat{r}_S = \hat{W}_S$, and manually fixing the entry of firms (as Chaney (2008) did), aggregate trade values reduce to

$$\hat{V}_{SI} = -\alpha \hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha \hat{\tau}_{SI}, \tag{86}$$

which replicates the elasticity result in Chaney (2008).

If we further allow for free entry and exit, and let the fixed cost respond to factor prices, then

$$\hat{Z}^{SI}_{\min} = \frac{1}{1-\sigma}\hat{B}_I + \frac{\sigma}{\sigma-1}\big(\xi \hat{W}_S + (1-\xi)\hat{r}\big) + \hat{\tau}_{SI}. \tag{87}$$

Substituting Equation (87) into Equation (84) yields

$$\hat{V}_{SI} = \hat{N}_S + \frac{\alpha}{1-\sigma}\hat{B}_I + \xi\left(1 - \frac{\sigma\alpha}{\sigma-1}\right)\hat{W}_S + (1-\xi)\left(1 - \frac{\sigma\alpha}{\sigma-1}\right)\hat{r} - \alpha\hat{\tau}_{SI}. \tag{88}$$

Denote $\frac{\sigma\alpha}{\sigma-1} \equiv \epsilon$. Since $\hat{N}_S = 0$, the wage elasticity is $\xi(1-\epsilon)$.

Finally, if capital is immobile, then $\hat{N}_S = (1-\xi)\hat{W}_S$, and the elasticity becomes

$$(1-\xi) + \xi - \xi\epsilon = 1 - \xi\epsilon,$$

reflecting the combined intensive- and extensive-margin responses.

F Matrix Form with $\hat{N}_J$ and $\hat{N}_L$

$$\begin{pmatrix}
1 & 0 & \xi - 1 & 0 & 0 & 0 \\
0 & 1 & 0 & \xi - 1 & 0 & 0 \\
\sum_I V_{JI} & 0 & (1-\xi(1-\epsilon))\sum_I V_{JI} & 0 & \frac{\alpha}{1-\sigma}V_{JJ} & \frac{\alpha}{1-\sigma}V_{JL} \\
0 & \sum_I V_{LI} & 0 & (1-\xi(1-\epsilon))\sum_I V_{LI} & \frac{\alpha}{1-\sigma}V_{LJ} & \frac{\alpha}{1-\sigma}V_{LL} \\
0 & 0 & \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\
-V_{JJ} & -V_{LJ} & \xi\sum_I V_{JI} - \xi(1-\epsilon)V_{JJ} & -\xi(1-\epsilon)V_{LJ} & \frac{\alpha}{1-\sigma}(V_{JJ}+V_{LJ}) & 0
\end{pmatrix}
\begin{pmatrix}
\hat{N}_J \\ \hat{N}_L \\ \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L
\end{pmatrix}
= \begin{pmatrix}
0 \\ 0 \\ -\alpha V_{JL}\hat{\tau}_{JL} \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -\alpha V_{LJ}\hat{\tau}_{LJ}
\end{pmatrix}$$

G Term $A > 0$

First, we evaluate the sign of A. It is reasonable to assume that the fixed cost for exporting is bigger than fixed cost for selling in the domestic market, and the iceberg cost will never be negative. Given these assumption, I have $\lambda < \frac{1}{2}$, i.e, the proportion of GDP from exporting is always less than 50%. Then I have $1 - \lambda > \frac{1}{2}$. So $A > (1-\xi) + 2\lambda\xi + 2\lambda(\epsilon\xi - 1) = (1-\xi) + 2\lambda\xi + 2\lambda\epsilon\xi - 2\lambda = (1-\xi) + 2\lambda(\xi - 1) + 2\lambda\epsilon\xi = (1-\xi)(1-2\lambda) + 2\lambda\epsilon\xi > 0$.

H Order of conditions

I will list these conditions again here:

First the condition for price index to increase with a unilateral trade liberalization, thus make investors lose welfare from the unilateral trade liberalization:

$$\lambda((1-2\lambda)(1-\epsilon\xi) - \xi) > \epsilon\xi \tag{89}$$

Then the condition for workers to lose welfare from the unilateral trade liberalization:

$$(1 + \frac{\xi}{\sigma - 1})\alpha(1-\lambda) + \lambda((1-2\lambda)(1-\epsilon\xi) - \xi) > \epsilon\xi \tag{90}$$

Finally the condition for export share of GDP to decrease with a unilateral trade liberalization:

$$(1 - 2\lambda(1-\epsilon\xi)) - \xi > \epsilon\xi \tag{91}$$

Given the common term one can see immediately that Condition 89 is stricter than Condition 90. Additionally, assume $\lambda < \frac{1}{2}$, it is also immediate that Condition 89 is stricter than Condition 91.

To decide the strictness between Condition 91 and Condition 90, I firstly assume that Condition 91 is satisfied, then I will prove that Condition 90 is automatically satisfied after.

Assume Condition 91 is satisfied gives:

$$\lambda((1-2\lambda)(1-\epsilon\xi) - \xi) > \lambda\epsilon\xi$$

Substitute the above term back to Condition 90, I get:

$$(1 + \frac{\xi}{\sigma - 1})\alpha(1-\lambda) + \lambda((1-2\lambda)(1-\epsilon\xi) - \xi) > (1 + \frac{\xi}{\sigma - 1})\alpha(1-\lambda) + \lambda\epsilon\xi$$

If I can prove that

$$(1+\frac{\xi}{\sigma-1})\alpha(1-\lambda)+\lambda\epsilon\xi > \epsilon\xi \tag{92}$$

Then I am done. To prove Condition 92 is true, I spread the terms including spreading ϵ as $\frac{\sigma\alpha}{\sigma-1}$ and I have:

$$(1+\frac{\xi}{\sigma-1})(\alpha-\alpha\lambda)+\lambda\frac{\sigma\alpha}{\sigma-1}\xi > \frac{\sigma\alpha}{\sigma-1}\xi$$

This is the same as proving:

$$\alpha-\alpha\lambda+\frac{\alpha\xi}{\sigma-1}-\frac{\alpha\lambda\xi}{\sigma-1}+\lambda\frac{\sigma\alpha}{\sigma-1}\xi-\frac{\sigma\alpha}{\sigma-1}\xi > 0$$

Re-arrange the terms gives:

$$\alpha(1-\lambda)+\frac{\alpha\xi}{\sigma-1}(1-\lambda+\lambda\sigma-\sigma) > 0$$

Re-arrange the terms further gives:

$$\begin{aligned}\alpha(1-\lambda)+\frac{\alpha\xi}{\sigma-1}((1-\lambda)+(\lambda-1)\sigma)\\ =\alpha(1-\lambda)+\frac{\alpha\xi}{\sigma-1}(1-\lambda)(1-\sigma)\\ =\alpha(1-\lambda)-\alpha\xi(1-\lambda)\\ =\alpha(1-\lambda)(1-\xi)>0\end{aligned} \tag{93}$$

So I proved that Condition 91 satisfied meaning Condition 90 automatically get satisfied, which means I can order these three conditions in there strictness as: Condition 90 is the loosest, Condition 91 is in the middle, and Condition 89 is the strictest.

I Calibrated Trade Cost Matrix and Regional Aggregate Fit

Calibrated result for F_{SI} matrix is as Figure 7.

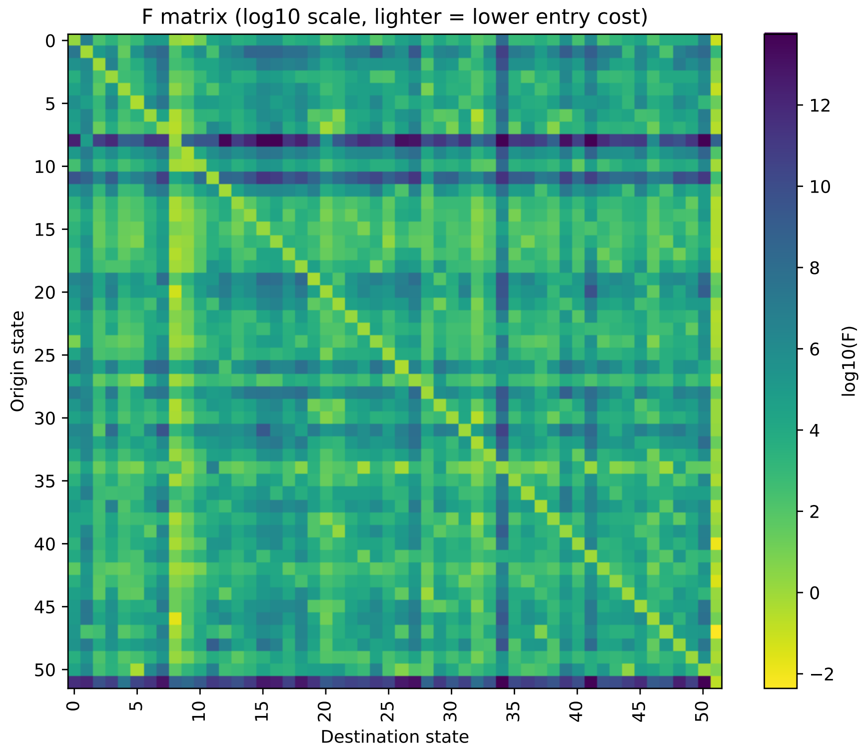

Figure 7: Model-implied versus data wage share moments (2017).

Figures 8–10 compare model-implied regional GDP, personal consumption expenditure (PCE), and wage compensation shares with their BEA counterparts. Across all three measures, the model closely reproduces the corresponding regional share distributions observed in the data.

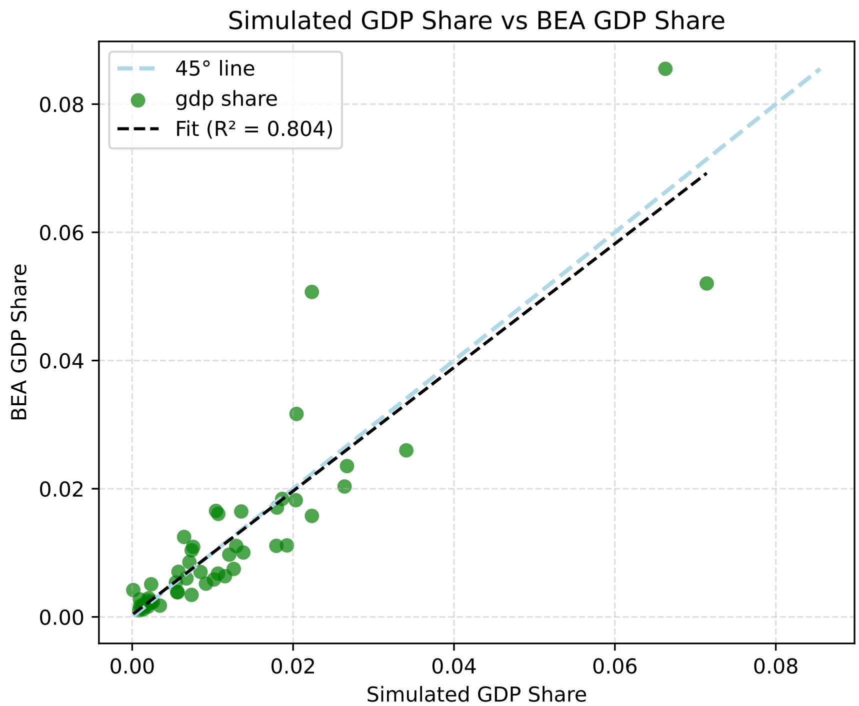

Figure 8: Model-implied versus BEA-reported regional GDP shares (2017).

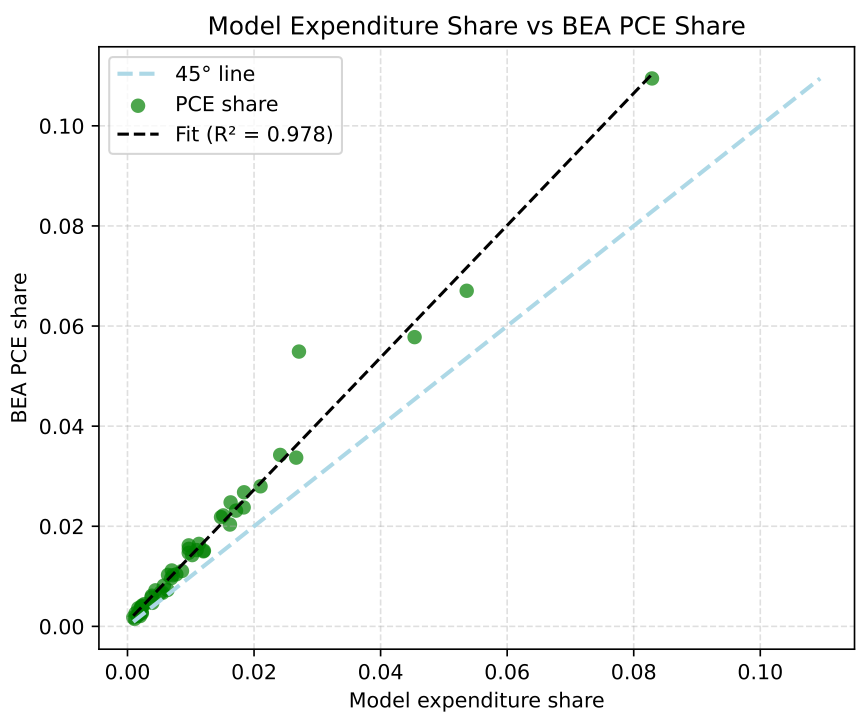

Figure 9: Model-implied versus BEA-reported regional personal consumption expenditure (PCE) shares (2017).

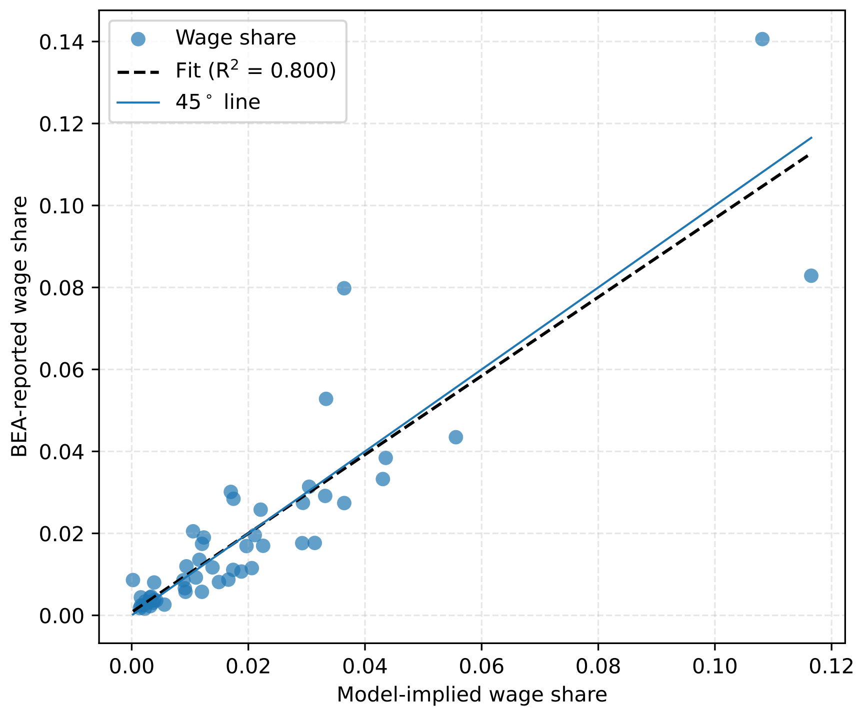

Figure 10: Model-implied versus data wage share moments (2017).

J Validate N_I

igging a bit deeper, I next validate whether the model-predicted percentage change in the number of firms aligns with the data-implied percentage change in the number of establishments following the tariff policy.

The most direct validation exercise is to test whether the model-predicted change in the number of firms, $\hat{N}_S^{\text{model}}$, is significantly positively correlated with the data-reported change, $\hat{N}_S^{\text{data}}$. Column (1) of Table 4 shows that this correlation is statistically insignificant. This null result is not surprising. Even if firm entry responds to wage changes as implied by the model, the tariff-induced component of the true wage response may be obscured by wage movements driven by non-tariff factors. As a result, the mapping between $\hat{N}_S^{\text{model}}$ and $\hat{N}_S^{\text{data}}$ can be weak in the data despite the structural relationship holding in the model.

Mathematically, this can be expressed as follows:

$$\hat{N}_S^{\text{model}} = g\Big(\hat{W}_S^{\text{model}}\Big) \approx (1-\xi)\,\hat{W}_S^{\text{model}} + \delta_S^N, \tag{94}$$

$$\hat{W}_S^* = \hat{W}_{\text{tariff},S}^* + \hat{W}_{\text{other},S}^*, \tag{95}$$

$$\hat{W}_S^{\text{model}} = \hat{W}_{\text{tariff},S}^* + \eta_S, \tag{96}$$

$$\hat{N}_S^{\text{data}} = g\Big(\hat{W}_S^*\Big) + \epsilon_S^N \approx (1-\xi)\,\hat{W}_S^* + \epsilon_S^N. \tag{97}$$

Substituting these expressions yields

$$\hat{N}_S^{\text{data}} \approx (1-\xi)\hat{W}_S^{\text{model}} + (1-\xi)\big(\hat{W}_{\text{other},S}^* - \eta_S\big) + \epsilon_S^N. \tag{98}$$

Therefore, even under a linear approximation, regressions of $\hat{N}_S^{\text{data}}$ on $\hat{N}_S^{\text{model}}$ or on $\hat{W}_S^{\text{model}}$ may yield weak or insignificant correlations when wage movements unrelated to tariffs and model-implied noise dominate the tariff-induced component of the true wage response, a difficulty that is further compounded when the underlying mapping is nonlinear and not invertible.

Unfortunately, unlike in the welfare case, I do not observe an alternative data proxy for the change in the number of firms that would allow me to net out non-tariff wage movements. I therefore focus on constructing a more accurate proxy for the tariff-induced component of the true wage change, $W_{\text{tariff},S}^*$, which in turn governs the model-implied response of firm entry.

From Equation 28, the model-predicted change in regional GDP is also tied to movements in the tariff-induced component of wages:

$$\widehat{\text{GDP}}_S^{\text{model}} = h\Big(\hat{W}_S^{\text{model}}\Big) \approx \hat{W}_S^{\text{model}} + \delta_S^{\text{GDP}}. \tag{99}$$

Because $\hat{N}_S^{\text{model}}$ and $\widehat{\text{GDP}}_S^{\text{model}}$ are distinct nonlinear transformations of the same underlying tariff-induced wage shock, the linear projection of $W_{\text{tariff},S}^*$ on $\big(\hat{N}_S^{\text{model}}, \widehat{\text{GDP}}_S^{\text{model}}\big)$ can be at least weakly more informative than the projection on $\hat{W}_S^{\text{model}}$ alone.

Note that this combination must exclude the direct proxy $\hat{W}_S^{\text{model}}$, since including it would lead the optimal linear combination to place all weight on $\hat{W}_S^{\text{model}}$, yielding no improvement[9].

Column (2) of Table 4 should be interpreted as a linear projection rather than a structural relationship. Since $\hat{N}_S^{\text{model}}$ and $\widehat{\text{GDP}}_S^{\text{model}}$ are both noisy, nonlinear functions of the same latent tariff-induced wage component, $\hat{W}_{\text{tariff},S}^*$, the coefficients should not be read as evidence that the model-implied change in the number of firms matches the observed change

[9]Check Appendix K for a detailed explanation

in establishments. Rather, the joint significance of the two regressors, together with the improvement in fit relative to Column (1), suggests that changes in establishments covary with the underlying tariff-induced wage response that the model is intended to capture. The opposite signs reflect partialling-out of shared variation and are not meaningful on their own.

Table 4: Tariff Number of Firms Validation(HC3 robust SEs)

	(1) %Δ #Firms (data)	(2) %Δ #Firms (data)
%Δ #firms (model)	1.651	4.619***
	(1.263)	(1.370)
%Δ GDP (model)		-1.202***
		(0.391)
Constant	-0.012	-0.024**
	(0.012)	(0.010)
Observations	51	51
R-squared	0.026	0.138
Adj. R-squared	0.006	0.102

K Constructing a Better Proxy

This appendix clarifies why including the direct wage proxy $\hat{W}_S^{\text{model}}$ alongside other model-implied variables that are deterministic functions of $\hat{W}_S^{\text{model}}$ cannot improve the proxy content in a linear regression.

Suppose the change in the number of firms observed in the data satisfies the (linearized) relationship

$$\hat{N}_S^{\text{data}} = \beta_1 \hat{W}_S^* + \epsilon_S^N, \tag{100}$$

where the true wage change can be decomposed as $\hat{W}_S^* = \hat{W}_{\text{tariff},S}^* + \hat{W}_{\text{other},S}^*$. The model-predicted wage change provides a noisy proxy for the tariff-induced component,

$$\hat{W}_S^{\text{model}} = \hat{W}_{\text{tariff},S}^* + \eta_S. \tag{101}$$

Moreover, under the linear approximation, the model-predicted change in the number of firms satisfies

$$\hat{N}_S^{\text{model}} \approx (1-\xi)\hat{W}_S^{\text{model}} + \delta_S^N, \tag{102}$$

so that

$$\hat{W}_S^{\text{model}} \approx \frac{1}{1-\xi}\Big(\hat{N}_S^{\text{model}} - \delta_S^N\Big). \tag{103}$$

Substituting (101) into (100) yields

$$\hat{N}_S^{\text{data}} = \beta_1 \hat{W}_S^{\text{model}} + \beta_1 \big(\hat{W}^*_{\text{other},S} - \eta_S\big) + \epsilon_S^N. \tag{104}$$

Equivalently, using (103), this can be rewritten in terms of $\hat{N}_S^{\text{model}}$ as

$$\hat{N}_S^{\text{data}} \approx \frac{\beta_1}{1-\xi} \hat{N}_S^{\text{model}} + \underbrace{\beta_1 \big(\hat{W}^*_{\text{other},S} - \eta_S\big) - \frac{\beta_1}{1-\xi} \delta_S^N + \epsilon_S^N}_{\text{composite error term}}. \tag{105}$$

Now consider augmenting this regression with exactly $\hat{W}_S^{\text{model}}$:

$$\hat{N}_S^{\text{data}} = \beta_1 \hat{N}_S^{\text{model}} + \beta_2 \hat{W}_S^{\text{model}} + e_S. \tag{106}$$

By the Frisch–Waugh–Lovell theorem, β_1 is identified from the variation in $\hat{N}_S^{\text{model}}$ that is orthogonal to $\hat{W}_S^{\text{model}}$:

$$\beta_1 = \frac{\operatorname{Cov}\!\big(\tilde{\hat{N}}_S^{\text{data}}, \tilde{\hat{N}}_S^{\text{model}}\big)}{\operatorname{Var}\!\big(\tilde{\hat{N}}_S^{\text{model}}\big)}, \qquad \tilde{\hat{N}}_S^{\text{model}} = \hat{N}_S^{\text{model}} - \mathbb{E}\Big[\hat{N}_S^{\text{model}} \mid \hat{W}_S^{\text{model}}\Big] .. \tag{107}$$

Because the model implies that $\hat{N}_S^{\text{model}}$ is a deterministic function of $\hat{W}_S^{\text{model}}$, i.e. $\hat{N}_S^{\text{model}} = g(\hat{W}_S^{\text{model}})$, we have

$$\mathbb{E}\Big[\hat{N}_S^{\text{model}} \mid \hat{W}_S^{\text{model}}\Big] = g(\hat{W}_S^{\text{model}}) = \hat{N}_S^{\text{model}}, \tag{108}$$

which implies

$$\tilde{\hat{N}}_S^{\text{model}} = 0. \tag{109}$$

Therefore, adding $\hat{W}_S^{\text{model}}$ on top of $\hat{N}_S^{\text{model}}$ provides no additional independent variation when $\hat{N}_S^{\text{model}}$ is a deterministic function of $\hat{W}_S^{\text{model}}$.

However, augmenting the proxy set with another model-predicted variable that is tied to wages but embeds a different approximation or measurement component—such as the model-predicted change in regional GDP—can in principle improve the approximation of the underlying tariff-induced wage shock in the sense of linear projection.

$$\hat{N}_S^{\text{data}} = \beta_1 \hat{N}_S^{\text{model}} + \beta_2 \hat{\text{GDP}}_S^{\text{model}} + e_S. \tag{110}$$

By the Frisch–Waugh–Lovell theorem, the coefficient β_1 is identified from the variation in

$\hat{N}_S^{\text{model}}$ that is orthogonal to $\widehat{\text{GDP}}_S^{\text{model}}$:

$$\beta_1 = \frac{\text{Cov}\big(\tilde{\hat{N}}_S^{\text{data}},\, \tilde{\hat{N}}_S^{\text{model}}\big)}{\text{Var}\big(\tilde{\hat{N}}_S^{\text{model}}\big)}, \qquad \tilde{\hat{N}}_S^{\text{model}} = \hat{N}_S^{\text{model}} - \mathbb{E}\Big[\hat{N}_S^{\text{model}} \mid \widehat{\text{GDP}}_S^{\text{model}}\Big]. \tag{111}$$

Analogously, β_2 is identified from the component of $\widehat{\text{GDP}}_S^{\text{model}}$ orthogonal to $\hat{N}_S^{\text{model}}$.

Unlike the case in which $\hat{W}_S^{\text{model}}$ is included directly, $\tilde{\hat{N}}_S^{\text{model}}$ need not be identically zero here. To see this, note that even under a linear approximation,

$$\hat{N}_S^{\text{model}} \approx (1-\xi)\hat{W}_S^{\text{model}} + \delta_S^N, \tag{112}$$

$$\widehat{\text{GDP}}_S^{\text{model}} \approx \hat{W}_S^{\text{model}} + \delta_S^{\text{GDP}}, \tag{113}$$

so that both variables share the same underlying $\hat{W}_S^{\text{model}}$ but differ in their residual components. Unless δ_S^N is perfectly collinear with δ_S^{GDP} (a knife-edge case), $\hat{N}_S^{\text{model}}$ is not fully spanned by $\widehat{\text{GDP}}_S^{\text{model}}$, and therefore $\tilde{\hat{N}}_S^{\text{model}} \neq 0$ in general.

Moreover, because the equilibrium mapping from tariffs to outcomes is nonlinear, the linear approximation omits higher-order terms that generally differ across outcomes. Consequently, the residual components δ_S^N and δ_S^{GDP} are generically not collinear, making the knife-edge condition under which $\tilde{\hat{N}}_S^{\text{model}} = 0$ even less likely to hold.

As a result, jointly using $\hat{N}_S^{\text{model}}$ and $\widehat{\text{GDP}}_S^{\text{model}}$ can weakly improve the linear projection approximation of the underlying tariff-induced wage component.