Globalization with Capital Integration and Spatial Deindustrialization

Dizhi Wang

April 29, 2025

This paper extends the canonical model of firm heterogeneity and trade by incorporating a mobile capital market and a global capital market condition that endogenously links wages and firm entry—an element absent in standard frameworks. The impact of this mechanism depends critically on the intensity of the mobile factor in production. When capital intensity is low, unilateral trade liberalization reduces both wages and prices, but the price-reducing effect is dampened due to firm delocation. As capital intensity increases, prices fall by less due to delocation effects from capital flight. At very high capital intensities, delocation effects dominate and unilateral liberalization leads to higher prices, producing outcomes reminiscent of the Metzler paradox. Calibrating the model to US statistics (0.33 capital share, 0.11 export share of GDP) workers experience welfare losses under unilateral tariff reductions, as wage declines outweigh price decreases. Overall, we find that countries may optimally raise unilateral trade barriers to prevent capital flight. These findings offer new insights into trade policy and the structural dynamics of deindustrialization.

Dizhi Wang: University of California, Santa Cruz

1. Introduction

Over the past few decades, many developed countries have experienced a widespread trend of manufacturing factories shutting down and relocating to developing countries—a phenomenon commonly referred to as *deindustrialization*. In the case of the United States, a growing body of research—most notably by Autor, Dorn, and Hanson (2013a)—has documented the adverse effects of the so-called "China shock" on domestic manufacturing, including declines in patenting, higher unemployment rates, lower wages, and sluggish job growth (Autor et al. 2020; Autor, Dorn, and Hanson 2013b; Acemoglu et al. 2016). Importantly, this trend is not limited to advanced economies; many developing countries have also experienced *premature deindustrialization*, as highlighted by Rodrik (2016). This paper develops a model that intuitively captures these dynamics and explores their potential welfare implications.

Factories in the real world and capital in economic models form an intuitive morphism, particularly when capital is interpreted as the accumulated embodiment of non-labor inputs. In this light, the phenomenon of factory relocation can be mapped to capital mobility in trade models. Perhaps due to path dependence, researchers often begin their analysis within the Heckscher–Ohlin (H-O) framework. However, a key limitation of the H-O model is that it attributes trade patterns to differences in countries' relative factor endowments and the factor intensities of the goods they produce. Empirical evidence suggests that this mechanism is not always central—particularly in the modern global economy, where national accounts across countries tend to reveal broadly similar factor shares, with labor typically accounting for approximately two-thirds and capital for one-third of value added. For this reason, I develop a model that begins with a framework in which trade is driven by a preference for variety.

In this paper, I extend the Melitz (2003) model by incorporating an integrated capital market and assigning ownership of capital to investors. This framework emphasizes the role of capital mobility in trade, as the movement of factories becomes a central focus. Separating consumers into investors who earn rental income and workers who learn salary income allows an exploration of the the potential unequal distribution of welfare under trade liberalization.

Under the symmetric-country case, I establish two key results: (1) bilateral trade liberalization does not induce capital relocation or alter wages, and it improves welfare for agents in both countries; (2) unilateral trade liberalization triggers capital outflows, resulting in a lower domestic wage, a contraction in the domestic firm mass, and reduced domestic production activity—outcomes that can negatively impact domestic welfare.

The extent of this welfare impact depends critically on the intensity of the mobile input (capital) in production. My numerical analysis in Section 4.3.2 shows that when capital intensity is below 0.3, the gains from lower prices dominate the losses from wage reductions, yielding net welfare improvements for both workers and investors. When capital intensity rises above 0.3 but remains moderate—for instance, at 0.33, as commonly reported in national accounts —investors still benefit from falling prices, but workers suffer welfare losses, as wage declines outweigh the gains from cheaper goods.

As capital intensity continues to increase, worker welfare deteriorates further, since the decline in the price index is no longer sufficient to compensate for wage suppression. Eventually, when capital intensity becomes extremely high—above 0.98—the contraction in domestic production is so severe that it reverses the price effect entirely: the aggregate price index rises, and both workers and investors incur welfare losses.

Positioning in the literature. This work contributes to the literature on capital flows in open economies. One strand of research examines capital mobility in pure endowment economies without production, such as Costinot, Lorenzoni, and Werning (2014) and Lloyd and Marin (2024). A second strand focuses on international finance, where Hu (2024) adopts the standard home-bias framework.

A third line of research introduces capital mobility into trade models. Beginning with Mundell (1957), a series of studies incorporate capital mobility into the Heckscher–Ohlin (H-O) framework, including Wong (1986), Kemp (1966), Jones (1967), Purvis (1972), Svensson (1984), Markusen and Svensson (1985), Foley and Manova (2014), and Edwards and van Wijnbergen (1986). Extensions to new trade theory include Martin and Rogers (1995) and Boulhol (2009), who embed capital mobility within the Krugman (1991) model, which features an outside sector (e.g., agriculture) that directly pins down the wage. Similarly, Kleinman et al. (2023) introduce capital mobility into the Armington (1969) framework.

Following this third line of research, I incorporate capital mobility into the Melitz (2003) framework. To highlight the novelty of the mechanism, the Appendix extends the Armington (1969) model to include capital mobility and demonstrates that the key mechanism emphasized in this paper does not arise due to the fixed-entry assumption embedded in the Armington (1969) framework. The same result holds for other fixed-entry frameworks, such as Eaton and Kortum (2002) and Chaney (2008).

With capital mobility, this model contributes to the classic question in international trade: are factor movements and commodity trade complements or substitutes? Mundell (1957) showed that under the standard H–O framework, the movement of goods and factors are substitutes. Subsequent work, including Wong (1986), Kemp (1966), Jones (1967), Purvis (1972), Svensson (1984), Markusen and Svensson (1985), and Foley and Manova (2014), has demonstrated that, in the absence of endowment composition differences across countries, commodity trade and factor movement tend to be substitutes. Krugman (1991) also showed that when labor is the only factor, labor movement substitutes for trade in goods. However, since labor is also the consumer, it must be treated as a special type of input. In this paper, I show that within the new trade theory framework, capital mobility can serve as a *complement* to commodity trade.

This work is also related to research that tries to explain why governments levy trade barriers. The most classic explanation is that governments seek to maintain favorable terms of trade, which means they want the price of their exported goods to be higher than the price of the goods they import, allowing them to purchase more imports with their export earnings. However, a higher export price does not necessarily equate to higher income. This explanation is only valid when the country can act as a monopolist exploiting an imperfectly elastic foreign supply of its imports or demand for its exports as Humphrey (1987) pointed out.

Furthermore, Caliendo et al. (2021) extend the Melitz model to incorporate roundabout production, showing that firm markups raise production costs and justify the use of optimal tariffs to correct the resulting distortions. Melitz and Ottaviano (2008) develop a model with quadratic utility—yielding linear demand—and find that unilateral trade liberalization induces firm relocation and welfare losses. This paper offers an alternative explanation: under a CES nested utility structure, governments may still have incentives to impose trade barriers—not to correct markups or linear-demand distortions, but to prevent the outflow of critical production factors, such as capital, in order to sustain domestic production activity.

Finally, this work relates to research examining the welfare losses from the China shock. Caliendo, Dvorkin, and Parro (2019) show that wage rigidities can lead to short-run welfare losses, while Harrison, McLaren, and McMillan (2011) highlight significant inequalities in the distribution of welfare gains across heterogeneous agents. In contrast, this paper shows that workers can lose welfare even in the absence of labor market frictions, such as job-switching costs or rigid wages. Moreover, by distinguishing between investors and consumers, it highlights how trade liberalization can generate divergent welfare outcomes for different types of domestic agents.

2. Model Setup

In this model, I have two countries: country J and country L. The total capital endowment in the world is K_0 and they are controlled by two representative capital owners: one lives in country J, one lives in L, i.e, $K_0 = K_J + K_L$, and $K_J = \gamma K_0, K_L = (1-\gamma)K_0$. The term γ determines the distribution of capital ownership that is exogenous in this static setup.

The total population in the two countries is denoted by L_J and L_L. The mass of workers in each country is $\mu_J L_J$ and $\mu_L L_L$, respectively, while the mass of investors is $(1-\mu_J)L_J$ and $(1-\mu_L)L_L$. The parameters $\{\mu_J, \mu_L, L_J, L_L\}$ are all exogeneous as well. I consider a scenario in which investors can allocate capital freely between the two countries without frictions.

2.1. Consumer Side

Both workers and investors at $I \in \{J, L\}$ are solving the similar problem

$$\max_{q_I(\omega)} U_I = \left(\int_{\omega\in\Omega_I} (q_I(\omega))^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

given their budge constraints as:

$$\int_{\omega\in\Omega_I} p_I(\omega) q_I(\omega) d\omega < Y,$$

where Y is the income, which is the salary income for workers, and rental income for investors. Solving the consumer problem yields the demand for goods ω from worker population be

$$q_I^w(\omega) = \frac{p_{SI}(\omega)^{-\sigma}}{P_I^{1-\sigma}} W_I \mu_I L_I,$$

and the demand from investors be:

$$q_I^m(\omega) = \frac{p_I(\omega)^{-\sigma}}{P_I^{1-\sigma}} r K_I$$

The total demand from consumers in country I is therefore given by

$$q_I(\omega) = q_I^w(\omega) + q_I^m(\omega) = \frac{p_I(\omega)^{-\sigma}}{P_I^{1-\sigma}} (r\gamma K_0 + W_I \mu_I L_I)$$

2.2. Firm Side

Each firm is uniquely identified with the goods ω they produce. Each firm also needs to draw a technology value from a continuous distribution function, and thus, they are also uniquely identified with their technology level z. Currently. I assume that there is only one sector where all firms production function follows the Cobb-Douglas form of

$$Y(\omega) = Z\left(\frac{l}{\xi}\right)^{\xi}\left(\frac{k}{1-\xi}\right)^{1-\xi}$$

The technology distribution function that a firm in country I draws from is

$$Z \sim \mathcal{F}^I(z) = \Pr(Z > z) = \begin{cases} (\frac{\underline{z}^I}{z})^{\alpha_I} & z > \underline{z}^I \\ 1 & \text{otherwise} \end{cases}$$

The demand function for variety ω from consumers in country I, denoted by $q_I(\omega)$, can be equivalently expressed as the demand for the product produced by firm z located in country S, denoted by $q_{SI}(z)$. Since firm productivity is drawn from a continuous distribution, the probability that two firms serving country I have the same productivity level is zero. As a result, including the source country S in the subscript does not impose any additional assumptions.

Firms minimize their unit production cost given the demand. For a firm at country S, the amount of capital and labor that they require to satisfy the demand from country I are

$$K_{S,I}(z) = \frac{1-\xi}{z}(\frac{W_S}{r})^{\xi} q_{SI}(z)(1+\tau_{SI})$$

$$L_{S,I}(z) = \frac{\xi}{z}(\frac{W_S}{r})^{\xi-1} q_{SI}(z)(1+\tau_{SI})$$

Finally, as a monopolistic competitive firm, it will set the price at the optimal level of

$$p_{S,I}(z) = \frac{\sigma}{\sigma-1} W_S^{\xi} r^{1-\xi} \frac{(1+\tau_{S,I})}{z}$$

2.3. Aggregate Values

The trade value from J to L for goods ω whose production technology is z is

$$V_{JL}(z) = \sigma B_L(1+\tau_{JL}) W_J^{\xi} r^{1-\xi} \frac{1}{z}$$

where

$$B_L = \frac{\mu_L W_L L_L + (1-\gamma) r K_0}{P_L^{1-\sigma}} (\frac{1}{\sigma})^{\sigma} (\frac{1}{\sigma-1})^{1-\sigma}$$

I interpret B_L as a scaled measure of "purchase size," as it increases with both price and income. Higher income implies greater demand for each variety, and higher aggregate prices reflect reduced competition across varieties, which also raises the demand faced by individual firms. Thus, a larger "purchase size" is associated with a more favorable environment for firm survival.

2.3.1. Technology Threshold for Export

For firms to export from J to L, they need to satisfy

$$\pi_{JL}(z) > F_{JL} W_J^{\xi} r^{1-\xi}$$

where F_{JL} is the fixed cost shifter for a firm from J to enter market L. Variable profits are derived as:

$$\pi_{JL}(z) = V_{JL}(z) - W_J L_{JL}(z) - rK_{JL}(z) = \frac{V_{JL}(z)}{\sigma}$$

Thus the productivity threshold is derived as:

$$z_{min}^{JL} = (\frac{F_{JL}}{B_L})^{\frac{1}{\sigma-1}} (W_J^{\xi} r^{1-\xi})^{\frac{\sigma}{\sigma-1}} (1 + \tau_{JL})$$

Furthermore, I will use $\tilde{\pi}_{JL}(z) = \pi_{JL}(z) - F_{JL} W_J^{\xi} r^{1-\xi}$ to represent the profits for firms after deducting the fixed cost for entering market L.

2.3.2. Aggregated Trade Value

Finally, the total trade value between J to L will be

$$V_{JL} = \int_z V_{JL}(z) dz = N_J \Pr(\tilde{\pi}_{JL} > 0) \mathbf{E}[V_{JL}(z) | \tilde{\pi}_{JL} > 0]$$

which gives:

$$V_{JL} = \frac{\sigma \alpha \underline{z}^{\alpha}}{\alpha + 1 - \sigma} N_J B_L^{\frac{\alpha}{\sigma-1}} F_{JL}^{1+\frac{\alpha}{1-\sigma}} (W_J^{\xi} r^{1-\xi})^{1-\epsilon} (1 + \tau_{JL})^{-\alpha} \tag{1}$$

where $\epsilon_J = \frac{\sigma \alpha_J}{\sigma - 1}$ and $\tilde{\pi}_{JL} = \int_z \tilde{\pi}_{JL}(z)\, dz = \int_z \left(\pi_{JL}(z) - F_{JL} W_J^{\xi} r^{1-\xi} \right) dz$ represent the total profit for all firms selling to country L after deducting total fixed costs. Equation 1 is fairly standard: a typical trade value equation derived under the Melitz (2003) framework with a Pareto productivity distribution would look almost identical, except that the term $W_J^{\xi} r^{1-\xi}$ would simply be W_J.

3. Equilibrium Conditions

This model features seven equilibrium conditions used to solve for six endogenous variables: $\{W_J, W_L, N_J, N_L, B_J, B_L\}$. The rental rate, r, does not appear in the list of endogenous variables because it is set as the numeraire. These conditions include two free entry conditions, two labor market clearing conditions, two trade balance conditions (one for each country), and one capital market clearing condition. Since there are only six variables, one condition can be removed—either a trade balance condition or a labor market clearing condition.

Free entry conditions require firms to operate with zero profits:

$$\sum_I \tilde{\pi}_{JI} = N_J F_J^E W_J^{\xi} r^{1-\xi}$$

Here, F_J^E denotes the shifter of fixed costs for firms operating in country J, and $\tilde{\pi}_{JI}$ is the profit of all firms operating in J after deducting fixed costs of entering market I. Given that

$$\sum_I \tilde{\pi}_{JI} = \frac{\sigma - 1}{\sigma\alpha} \sum_I V_{JI}, \tag{2}$$

the two free entry conditions can be simplified as:

$$\frac{\sigma - 1}{\sigma\alpha} \sum_I V_{JI} = N_J F_J^E W_J^{\xi} r^{1-\xi}, \tag{3}$$

$$\frac{\sigma - 1}{\sigma\alpha} \sum_I V_{LI} = N_L F_L^E W_L^{\xi} r^{1-\xi}. \tag{4}$$

Labor market clearing conditions require all workers to participate in production of either the goods or the fixed cost:

$$L_J = \tilde{L}_{JL} + \tilde{L}_{JJ} + N_J F_J^E \xi (\frac{W_J}{r})^{\xi - 1}$$

where $\tilde{L}_{JI}$ added the labor input for producing fixed cost of entering market I such that $\tilde{L}_{JI} = \xi \frac{V_{JI}}{\sigma W_J}(\sigma - 1 + \frac{\alpha + 1 - \sigma}{\alpha})$. Thus the final expression of labor market clearing condition is:

$$\mu_J L_J = \frac{\xi}{W_J} \sum_I V_{JI} \tag{5}$$

$$\mu_L L_L = \frac{\xi}{W_L} \sum_I V_{LI} \tag{6}$$

With the frictionless capital assumption, there is a single capital market, and its clearing condition requires that all capital is allocated either to production or to cover fixed costs. For country J, this includes capital used for producing consumption goods (K_{JI}), capital used to pay the fixed costs of selling in market I, and capital required for the fixed cost of operating in market J. Defining $\tilde{K}_{JI}$ as the sum of K_{JI} and the capital used for selling in market I, and using analogous notation for country L, the total capital market clearing condition is given by:

$$K_0 = \sum_I \tilde{K}_{JI} + \sum_I \tilde{K}_{LI} + N_J F_E^J (1-\xi)\left(\frac{W_J}{r}\right)^{\xi} + N_L F_E^L (1-\xi)\left(\frac{W_L}{r}\right)^{\xi} \tag{7}$$

where

$$\tilde{K}_{JI} = \frac{(1-\xi)}{\sigma r} V_{JI}\left(\sigma - 1 + \frac{\alpha + 1 - \sigma}{\alpha}\right) \tag{8}$$

Finally, the capital market clearing condition is:

$$K_0 = \frac{1-\xi}{r} \sum_I (V_{JI} + V_{LI}) \tag{9}$$

Combine equation (1) (2) (3) (4) together, we have:

$$W_J L_J + W_L L_L + rK_0 = \xi(\sum_I V_{JI} + \sum_I V_{LI}) + (1-\xi)(\sum_I V_{JI} + \sum_I V_{LI}) = \sum_I V_{JI} + \sum_I V_{LI} \tag{10}$$

which says that the total expenditure in this world will be equal to the total income.

Trade balance conditions require the total expenditures equal to the total income within each country thus I have:

$$\mu_J W_J L_J + \gamma r K_0 = V_{JJ} + V_{LJ} \tag{11}$$

$$\mu_L W_L L_L + (1-\gamma) r K_0 = V_{LL} + V_{JL} \tag{12}$$

The solution of the equilibrium conditions will yield the vector $\{W_J, W_L, B_J, B_L, N_J, N_L\}$ where the rental price r is the numéraire.

In the Appendix D, I prove that when the two countries are symmetric, solving the system of Equations 3, 4, 5, 6, 9, and 11, I can get a unique equilibrium solution with all variables being the same, i.e

$$V_{JL} = V_{LJ} \tag{13}$$

$$V_{JJ} = V_{LL} \tag{14}$$

$$(15) \qquad W_J = W_L$$

$$(16) \qquad B_J = B_L$$

$$(17) \qquad N_J = N_L$$

This equilibrium serves as the baseline from which counterfactual scenarios are evaluated.

4. Comparative Statics

In this section, I log-differentiate the equilibrium conditions around the symmetric equilibrium derived in the previous section. My primary goal is to analyze how an exogenous change in trade costs influences equilibrium outcomes.

Firstly, I combine the labor market clearing conditions (Equations 5 and 6) with the free entry conditions (Equations 3 and 4) to reduce the set of endogenous variables to four: $\{W_J, W_L, B_J, B_L\}$. This reduction is feasible because the mass of firms becomes a direct function of wages and parameters, including population and the fixed cost shifter.

$$(18) \qquad N_J = \frac{W_J^{1-\xi} r^{\xi-1} L_J}{\xi F_J^E} \frac{\sigma - 1}{\sigma\alpha},$$

$$(19) \qquad N_L = \frac{W_L^{1-\xi} r^{\xi-1} L_L}{\xi F_L^E} \frac{\sigma - 1}{\sigma\alpha}.$$

This linkage between the mass of firms and wages is new. In the standard Melitz (2003) framework, where $\xi = 1$, the mass of firms depends only on L_J and model parameters. In contrast, when $\xi < 1$, as in this setting, firm entry becomes endogenously linked to wages.

Log-differentiating Equations 18 and 19 yields:

$$(20) \qquad \hat{N}_J = (1 - \xi)\hat{W}_J$$

$$(21) \qquad \hat{N}_L = (1 - \xi)\hat{W}_L.$$

Log-differentiating the trade value equations (Equation 1) yields:

$$(22) \qquad \hat{V}_{JI} = (1 - \xi\epsilon)\hat{W}_J + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha\hat{\tau}_{JI}.$$

Similarly, log-differentiating the labor market clearing conditions gives:

$$\sum_I V_{JI}\hat{W}_J = \sum_I V_{JI}\hat{V}_{JI}, \tag{23}$$

$$\sum_I V_{LI}\hat{W}_L = \sum_I V_{LI}\hat{V}_{LI}. \tag{24}$$

Substituting Equation 22 into the log-linearized labor market clearing conditions above, I have:

$$\sum_I V_{JI}\hat{W}_J = (1-\xi\epsilon)\sum_I V_{JI}\hat{W}_J + V_{JJ}\frac{\alpha}{\sigma-1}\hat{B}_J + V_{JL}\frac{\alpha}{\sigma-1}\hat{B}_L - \alpha V_{JL}\hat{\tau}_{JL}, \tag{25}$$

and

$$\sum_I V_{LI}\hat{W}_L = (1-\xi\epsilon)\sum_I V_{LI}\hat{W}_L + V_{LJ}\frac{\alpha}{\sigma-1}\hat{B}_L + V_{LL}\frac{\alpha}{\sigma-1}\hat{B}_L - \alpha V_{LJ}\hat{\tau}_{LJ}. \tag{26}$$

Log-linearize capital clearing condition as Equation 9 gives:

$$\sum_I V_{JI}\hat{V}_{JI} + \sum_I V_{LI}\hat{V}_{LI} = 0 \tag{27}$$

Use Equation 23 and Equation 24, I can transform Equation 27 as:

$$\sum_I V_{JI}\hat{W}_J + \sum_I V_{LI}\hat{W}_L = 0 \tag{28}$$

The capital clearing condition thus restricted that the wages of the two countries must go to the opposite directions.

Note that summing the two labor market clearing conditions (Equations 5 and 6) and the two trade balance conditions (Equations 11 and 12) reproduces the capital market clearing condition (Equation 9). Due to this linear dependency, one equation must be omitted when solving for equilibrium or performing the log-differentiation. In the analysis that follows, I omit the trade balance condition for country L.

Log-differentiating the trade balance condition for country J yields:

$$W_J L_J \hat{W}_J = V_{JJ}\hat{V}_{JJ} + V_{LJ}\hat{V}_{LJ}.$$

Substituting Equation 22 into this expression gives:

$$(\xi \sum_{I} V_{JI} + V_{JJ}(\xi\epsilon - 1))\hat{W}_J + (\xi\epsilon - 1)V_{LJ}\hat{W}_L + \frac{\alpha(V_{JJ} + V_{LJ})}{1-\sigma}\hat{B}_J = -\alpha V_{LJ}\hat{\tau}_{LJ}. \tag{29}$$

The results of the log-linearization can be expressed in the following general matrix form. In this representation, $\hat{N}_J$ and $\hat{N}_L$ have been substituted with $\hat{W}_J$ and $\hat{W}_L$ using Equations 20 and 21. The original matrix formulation, which includes $\hat{N}_J$ and $\hat{N}_L$, is provided in Appendix A.

$$\begin{pmatrix} \xi\epsilon \sum_I V_{JI} & 0 & \frac{\alpha}{1-\sigma}V_{JJ} & \frac{\alpha}{1-\sigma}V_{JL} \\ 0 & \xi\epsilon \sum_I V_{LI} & \frac{\alpha}{1-\sigma}V_{LJ} & \frac{\alpha}{1-\sigma}V_{LL} \\ \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\ \xi \sum_I V_{JI} + (\xi\epsilon - 1)V_{JJ} & (\xi\epsilon - 1)V_{LJ} & \frac{\alpha}{1-\sigma}(V_{JJ} + V_{LJ}) & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha V_{JL}\hat{\tau}_{JL} \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \end{pmatrix}$$

I further simplify the expression by defining $\lambda = \frac{V_{JL}}{V_{JL}+V_{JJ}}$ as the share of GDP derived from exports. Using the equilibrium solutions for the symmetric case from Section 3, I obtain:

$$\begin{pmatrix} \xi\epsilon & 0 & \frac{\alpha(1-\lambda)}{1-\sigma} & \frac{\alpha}{1-\sigma}\lambda \\ 0 & \xi\epsilon & \frac{\alpha\lambda}{1-\sigma} & \frac{\alpha}{1-\sigma}(1-\lambda) \\ 1 & 1 & 0 & 0 \\ \xi + (\xi\epsilon - 1)(1-\lambda) & (\xi\epsilon - 1)\lambda & \frac{\alpha}{1-\sigma} & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha\lambda\hat{\tau}_{JL} \\ -\alpha\lambda\hat{\tau}_{LJ} \\ 0 \\ -\alpha\lambda\hat{\tau}_{LJ} \end{pmatrix}$$

4.1. Bilateral Trade Liberalization

To start the analysis, I analyze the effects of a symmetric reduction in trade costs between two countries, J and L. Suppose both countries reduce their trade barriers by the same amount:

$$\hat{\tau}_{JL} = \hat{\tau}_{LJ} = \hat{\tau} < 0.$$

Solving the system of equations, I obtain:

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ (\sigma - 1)\lambda\hat{\tau} \\ (\sigma - 1)\lambda\hat{\tau} \end{pmatrix}.$$

This result implies that **bilateral trade liberalization from a symmetric equilibrium does not affect the relative wages** in either country compared to the rental price. Consequently, the nominal income of all agents in both countries remains unchanged. However, trade liberalization alters the terms B_J and B_L by modifying the price index, and the change in the price index is identical for both countries. This outcome is consistent with the findings in Appendix F, where I examined the same scenario in the standard Melitz (2003) without capital mobility.

PROPOSITION 1. *Under bilateral trade liberalization, the real wages of workers in country $I \in \{J, L\}$ and the real rental income of investors increase the same amount by:*

$$\hat{W}_I - \hat{P}_I = -\hat{P}_I = \frac{-1}{\sigma - 1}(\hat{B}_I - s\hat{W}_I) = -\lambda\hat{\tau} > 0.$$

where s represents the labor income ratio:

$$s = \frac{\mu_J L_J W_J}{u_J L_J W_J + \gamma r K_0}.$$

While workers' nominal wages remain unchanged, they benefit from lower price levels following trade liberalization, resulting in an increase in real wages. Similarly, since the rental price r serves as the numeraire, investors' nominal income also remains constant. They too benefit from the reduction in the price index, which improves their real purchasing power.

4.2. Unilateral trade liberalization

When only my country (country J) reduces trade barriers for the foreign country (country L), the trade cost for goods traveling from country L to country J decreases, while the trade cost from country J to country L remains unchanged. That is, $\hat{\tau}_{JL} = 0$ and $\hat{\tau}_{LJ} = \hat{\tau} < 0$ (a unilateral trade cost reduction). Solving the system of equations yields:

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} \frac{\alpha\lambda(1-\lambda)}{A}\hat{\tau} \\ -\frac{\alpha\lambda(1-\lambda)}{A}\hat{\tau} \\ (\sigma-1)\frac{\lambda C}{A}\hat{\tau} \\ -(\sigma-1)\frac{(1-\lambda)D}{A}\hat{\tau} \end{pmatrix}$$

Where

$$A = (1-\xi) + 2\lambda\xi + 4\lambda(1-\lambda)(\epsilon\xi - 1) > 0$$

$$C = \lambda(\xi - (1-2\lambda)(1-\epsilon\xi)) + \epsilon\xi$$

$$D = \lambda(\xi - (1-2\lambda)(1-\epsilon\xi))$$

PROPOSITION 2. *Under unilateral trade liberalization, when the export share of GDP, λ, is less than $\frac{1}{2}$, nominal wages in country J decrease, while those in country L increase by the same magnitude.*

PROOF. Since $\lambda < 1$ (as it represents the export share of GDP), the sign of the change in nominal wages in country J is determined by the sign of the term A. As shown in Appendix B, the term A is positive when $\lambda < \frac{1}{2}$—a condition that is empirically reasonable. Therefore, wages in country J decrease, while those in country L rise by an equal amount. □

At the same time, since r is set as the numeraire, the nominal income of investors remains unchanged. Consequently, the welfare of domestic investors depends on the change in the domestic price index. With wage changes, the change of domestic price index will be:

$$\hat{P}_I = \frac{1}{\sigma-1}(\hat{B}_I - s\hat{W}_I) \tag{30}$$

For domestic country J, the price index change is thus:

$$\hat{P}_I = \frac{1}{\sigma-1}(\frac{(\sigma-1)\lambda C}{A} - s\frac{\alpha\lambda(1-\lambda)}{A})\hat{\tau} = \frac{\lambda}{A}(C - s\frac{\alpha(1-\lambda)}{(\sigma-1)})\hat{\tau} \tag{31}$$

Here, I want to discuss the difference between $\mu_J W_J L_J + \gamma r K_0$ and $V_{JJ} + V_{JL}$. Typically, both can be interpreted as GDP: $\mu_J W_J L_J + \gamma r K_0$ represents GDP from the income (or expenditure) perspective, while $V_{JJ} + V_{JL}$ represents GDP from the production perspective (which is directly the definition of GDP).

In a traditional trade model, these two measures are equal. However, in this setting, investors earn income from foreign as well as domestic production. By definition, this means the two GDP measures will not be the same when the two countries are not symmetric. In the symmetric case, where they are equal, s coincides with ξ, the labor income share of GDP.

Thus the change of domestic price index becomes:

$$\hat{P}_I = \frac{\lambda}{A}\left(C - \xi\frac{\alpha(1-\lambda)}{(\sigma-1)}\right)\hat{\tau} = \frac{\lambda}{A}F\hat{\tau} \tag{32}$$

When $F > 0$, a unilateral tariff reduction leads to a decrease in the domestic price index, resulting in welfare gains for investors. In contrast, when $F < 0$, the domestic price index rises following the tariff reduction—as in the Metzler paradox—and investors experience welfare losses.

The intuition behind the term F is as follows. It consists of two components: C, which captures the change in "purchase size," and $\xi\frac{\alpha(1-\lambda)}{\sigma-1}$, which reflects the change in wages. As discussed earlier, "purchase size" is positively related to both consumers' nominal income and the domestic price index. Proposition 2 shows that wages always decline under unilateral trade liberalization. Given that the rental price is the numéraire, consumers' nominal income necessarily falls. Therefore, any observed increase in "purchase size" must be entirely attributed to a rise in the domestic price index. Moreover, because nominal income decreases, the increase in the price index must first offset the income decline before it can contribute to an increase in "purchase size."

The condition for $F < 0$ is equivalent to:

$$\lambda((1-2\lambda)(1-\epsilon\xi)-\xi) + \xi\frac{\alpha(1-\lambda)}{(\sigma-1)} > \epsilon\xi \tag{33}$$

The sufficient condition for $F < 0$ is $C < 0$, as $F = C - \xi\frac{\alpha(1-\lambda)}{\sigma-1}$ and $\xi\frac{\alpha(1-\lambda)}{\sigma-1} > 0$. Then I have:

$$C < 0 \iff \lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big) > \epsilon\xi \tag{34}$$

PROPOSITION 3. *There exists values of ξ such that $C < 0$ can be satisfied.*

PROOF. The proof will be an application of the Intermediate Value Theorem. The left-hand side of the inequality is strictly decreasing in ξ, while the right-hand side is strictly increasing. At $\xi = 0$, the inequality holds since the left-hand side is positive and the right-hand side is zero. At $\xi = 1$, the left-hand side is negative and the right-hand side is positive. By the Intermediate Value Theorem, there exists a unique threshold $\xi^* \in (0,1)$ such that the inequality reverses. Therefore, $C < 0$ holds if and only if $\xi < \xi^*$. □

PROPOSITION 4. *There exists values of ξ such that $F < 0$ can be satisfied*

PROOF. As $C < 0$ is the sufficient condition for $F < 0$, and given Proposition 3 holds, Proposition 4 holds. $\square$

In general, the smaller the values of ϵ and ξ, the more likely it is that $F < 0$, implying a greater likelihood that the domestic price index will increase with unilateral trade liberalization.

For workers, welfare is determined by the real wage, and the change in their welfare is given by:

$$\hat{W}_J - \hat{P}_J = \hat{W}_J - \frac{\hat{B}_J - s\hat{W}_J}{\sigma - 1} = \left(1 + \frac{\xi}{\sigma - 1}\right)\hat{W}_J - \frac{\hat{B}_J}{\sigma - 1}, \tag{35}$$

For workers to lose welfare from a unilateral trade liberalization, the following condition needed to be satisfied:

$$(1 + \frac{\xi}{\sigma - 1})\alpha(1 - \lambda) + \lambda((1 - 2\lambda)(1 - \epsilon\xi) - \xi) > \epsilon\xi \tag{36}$$

PROPOSITION 5. *Domestic workers are more susceptible to welfare losses from international trade compared to investors. There exist scenarios in which investors benefit from trade liberalization while workers experience welfare losses.*

PROOF. Condition 36 is less restrictive than Condition 33 since

$$\alpha(1 - \lambda) > 0$$

It follows that the set of values satisfying Condition 36 includes all values satisfying Condition 33 but also additional values where Condition 33 does not hold. $\square$

Finally, I want to analytically explore the relationship between the change of export share of GDP, $\hat{\lambda}$, and $\hat{\tau}_{LJ}$. Log-differentiating $\lambda = \frac{V_{JL}}{V_{JJ}+V_{JL}}$ under unilateral trade liberalization (i.e., $\hat{\tau}_{JL} = 0$) gives:

$$\hat{\lambda} = (1 - \lambda)(\hat{V}_{JL} - \hat{V}_{JJ}) = (1 - \lambda)\frac{\alpha}{\sigma - 1}(\hat{B}_L - \hat{B}_J) \tag{37}$$

Substituting $\hat{V}_{JJ}$ from Equation 22, I obtain:

$$\hat{\lambda} = -\frac{\alpha\lambda(1 - \lambda)\hat{\tau}}{A}(\xi - (1 - 2\lambda)(1 - \epsilon\xi) + \epsilon\xi). \tag{38}$$

The condition for $\hat{\lambda}$ to increase with a unilateral tariff increase (or decrease with a unilateral trade liberalization increase) is thus:

$$(1 - 2\lambda)(1 - \epsilon\xi) - \xi > \epsilon\xi. \tag{39}$$

PROPOSITION 6. *If a reduction in tariffs by country J ($\hat{\tau}_{LJ} = \hat{\tau} < 0$) leads to an increase in the price index of country J, then the domestic export share of GDP, λ, must decrease in response to this tariff reduction.*

PROOF. Assume Condition 33 holds, Condition 39 will hold. So Condition 33 is stricter than Condition 39. □

PROPOSITION 7. *Condition 33 ⇒ Condition 39 ⇒ Condition 36*

PROOF. Proof can be found in Appendix C. □

Proposition 7 demonstrates that as ξ decreases from 1 to 0, the economy sequentially transitions through the following scenarios. Initially, the reduction in the price index exceeds the decline in wages, so both domestic workers and investors experience welfare gains. As ξ falls further, workers in country *J* begin to incur welfare losses, as the price reduction no longer offsets the wage decline. During this phase, the export share of GDP, λ, continues to rise, and the domestic price index remains low—implying that investors still gain welfare.

As ξ decreases further, λ begins to decline under unilateral liberalization, though investors continue to benefit from lower aggregate prices. When ξ becomes sufficiently small, the domestic price index eventually rises in response to the tariff cut, producing a result reminiscent of the Metzler paradox.

Let us pause here and evaluate what the price index implies. Most of the time, this index is assessed from the consumers' perspective, but here, let's examine it from the producers' perspective. The price index represents an aggregation of the prices of all goods sold in a country. A lower price index implies that goods in the market are cheaper, which means a lower production cost is required for firms to survive. Consequently, a lower price index indicates a more competitive market.

When the value of ξ is sufficiently large, the familiar classic price effect emerges for a unilateral trade liberalization: domestic wages and the domestic price index decrease, while foreign wages and the foreign price index increase. In this case, the domestic market becomes more competitive, but domestic producers gain an advantage since the removal of unilateral trade barriers reduces wages, thereby lowering production costs. Conversely, foreign producers face a disadvantage as foreign wages increase.

Moreover, the increase in the foreign price index implies that the foreign market becomes easier to enter. With the wage advantage, domestic producers have a greater chance of surviving in the foreign market as well. This explains why λ increases with unilateral trade liberalization. From this perspective, workers somewhat are "transferring" part of their welfare (via lower wages) to producers, allowing domestic firms to gain a competitive edge. As long as the reduction in the price index offsets the wage loss, workers still benefit overall.

When ξ becomes smaller such that it now satisfies the loosest Condition 36 but not Condition 39 or Condition 33, the price index follows the same pattern: the domestic market becomes more difficult to compete in, while the foreign market becomes easier to enter. However, since labor becomes a less important factor in production, domestic producers must cut wages further (or receive greater transfers from workers) to survive in this more competitive market.

Even though labor's role diminishes, domestic producers can still enhance their competitiveness by reducing wage costs, ensuring that λ continues to increase with liberalization. However, the wage cuts will be substantial enough to offset the benefits of cheaper goods for workers. This can also be interpreted as a greater portion of workers' welfare being sacrificed to maintain the competitiveness of domestic producers, leading to a welfare loss for workers.

If ξ becomes even smaller, such that both Condition 36 and Condition 39 are satisfied, the domestic market continues to become more difficult to compete in, while the foreign market becomes increasingly accessible. However, the wage advantage gained from lowering domestic wages becomes so small that domestic producers can no longer maintain their competitiveness against foreign producers, causing λ to decrease with trade liberalization.

To survive, domestic producers must further reduce wages, requiring an even greater sacrifice of workers' welfare. However, these sacrifices are insufficient to sustain the competitiveness of domestic producers.

Finally, if ξ becomes extremely small, Condition 33 is satisfied. As a result, the foreign price index decreases while the domestic price index increases, meaning that the domestic market becomes easier to survive, while the foreign market becomes more difficult to compete in.

In this case, more foreign producers enter the domestic market. However, because labor is too insignificant a factor for domestic firms to retain a competitive advantage, domestic firms rapidly exit. As a result, an increasing share of goods is now supplied by foreign firms, which naturally charge higher prices due to iceberg trade costs and the fixed costs of exporting. Consequently, the domestic price index rises.

4.2.1. Understand the results through the wage elasticity

The key reason why introducing mobile capital generates such distinct results can be seen more clearly through the log-change of GDP in response to a unilateral tariff change:

$$\hat{V} = \widehat{(V_{JJ} + V_{JL})} = (1-\lambda)\hat{V}_{JJ} + \lambda\hat{V}_{JL} = (1-\xi\epsilon)\hat{W}_J + \frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J + \frac{\alpha\lambda}{\sigma}\hat{B}_L. \tag{40}$$

From this expression, it follows that smaller values of ϵ and σ result in a higher positive elasticity of wages with respect to GDP. However, in the canonical trade model, the elasticity of wages with respect to GDP is typically negative.

Chaney (2008) demonstrated that within the Melitz (2003) framework, assuming a Pareto distribution for firms' productivity and no firm entry or exit, the elasticity takes the form:

$$-\alpha < 0.$$

If firm entry and exit are allowed within the Chaney (2008) framework, the elasticity becomes:

$$1-\epsilon < 0.$$

I also conduct a similar welfare analysis under the Chaney (2008) framework allow firm entry and exit in Appendix F.

Likewise, in an Armington-style model, the elasticity is given by:

$$1-\sigma < 0.$$

This highlights a fundamental difference between this model and canonical frameworks, where capital mobility alters the wage-GDP relationship in ways that are not observed in standard models. Furthermore, the elasticity becomes $1-\epsilon\xi$ because, in this model, the mass of firms responds to changes in wages, as shown in Equation 20.

Going back to Equation 1, ignoring changes in the mass of firms, the elasticity of wages with respect to trade values is given by $\xi(1-\epsilon) < 0$. However, since the mass of firms now *responds* to wages, the full elasticity becomes $(1-\xi) + \xi - \xi\epsilon = 1 - \xi\epsilon$, reflecting both the effects of wages on the intensive and extensive margins.

When ϵ and ξ are sufficiently large such that $1-\epsilon\xi < 0$, the intensive margin effect dominates the extensive margin effect. Few firms exit the market, and the reduction in production costs for surviving firms boosts domestic output. As noted earlier, one can interpret the log-differentiated results as workers sacrificing or transferring welfare to support domestic producers' competitiveness in a more challenging market environment. When $1 - \epsilon\xi < 0$, this sacrifice is relatively efficient, as wage reductions translate into a meaningful increase in domestic production.

In contrast, when $1-\epsilon\xi > 0$, the extensive margin effect outweighs the intensive margin effect. Although surviving firms benefit from lower production costs, too few remain in the market to sustain output. In other words, labor becomes so insignificant that the sacrifice of workers' wages no longer contributes effectively to domestic production.

In summary, unlike bilateral trade liberalization—which does not affect the market share of each country in a perfectly symmetric scenario (i.e., both countries continue to produce 50% of the goods)—unilateral trade liberalization alters this balance. When country J (my country) unilaterally reduces trade barriers, the mass of firms in my country ($\hat{N}_J = (1 - \xi)\hat{W}_J$) decreases, while the mass of firms in the foreign country ($\hat{N}_L = (1 - \xi)\hat{W}_L$) increases.

The increase in production activity in the foreign country L attracts capital flows from J to L. However, if labor intensity is high (i.e., ξ is large), the decline in the mass of domestic firms will be limited, and the resulting wage reduction will be fully offset by a decline in the price index. Conversely, if labor intensity is low (i.e., ξ is small), capital outflows will lead to a substantial contraction in the domestic firm mass, resulting in significantly lower wages and a reduced likelihood of benefiting from lower prices.

4.3. Numeric Analysis

To visualize the results, I conduct a numerical analysis calibrated using parameter values drawn from previous studies, as summarized in Table 1. Following Bai, Jin, and Lu (2024), I adopt their values for F^E_J, F^E_L, L_J, and L_L. I then adjust the bilateral trade costs (F_{JL} and F_{LJ}), as well as the domestic selling costs (F_{JJ} and F_{LL}), to match the observed U.S. export-to-GDP ratio of approximately 11%.

Parameter	Value
σ	2
$\underline{Z}$	1
$\alpha_J = \alpha_L$	2
K_0	1
$\mu_J = \mu_L$	0.9
$L_J = L_L$	1.1
γ	0.5
$K_J = K_L$	$\gamma \cdot K_0 = 0.5$
$F_{JJ} = F_{LL}$	0.01
$F_{JL} = F_{LJ}$	0.08
$F_J^E = F_L^E$	1

TABLE 1. Parameter values for the model.

Given parameters as in Table 1, simulation result shows that export share of GDP λ for both country J and country L will be around 11% in the equilibrium. Substitute the values of λ and the values of parameters σ and α in Table 1, I can visualize the range of ξ that satisfy Condition 33, Condition 39, and Condition 36.

To satisfy Condition 33, I have:

$$0.11((1-2\times 0.11)(1-4\xi)-\xi)+\xi\frac{(1-0.11)}{(2-1)} > 4\xi \iff \xi < 0.025 \tag{41}$$

To satisfy Condition 39, I have:

$$(1-2\times 0.11)(1-4\xi)-\xi > 4\xi \iff \xi < 0.096 \tag{42}$$

To satisfy Condition 36, I have:

$$(1+\frac{\xi}{2-1})2(1-0.11)+0.11[(1-2\times 0.11)(1-4\xi)-\xi] > 4\xi \iff \xi < 0.697 \tag{43}$$

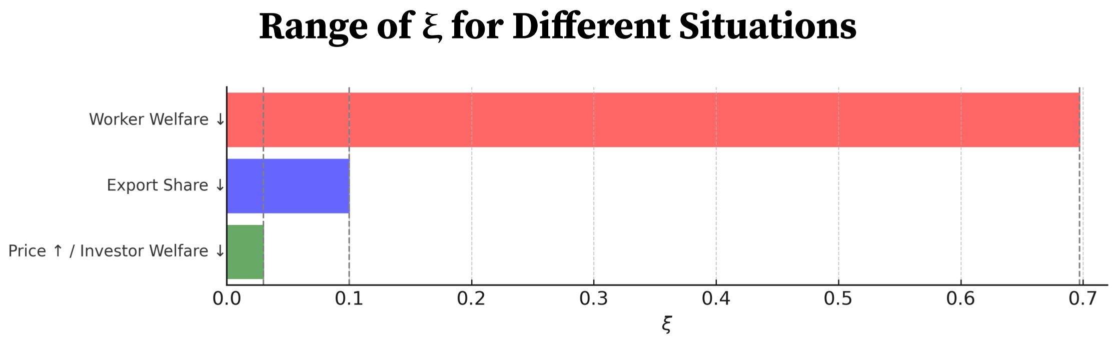

FIGURE 1. This figure shows the threshold values of the labor intensity parameter ξ under which various outcomes are triggered. The red bar corresponds to the range where worker welfare declines (Condition 36), the blue bar where export share declines (Condition 39), and the green bar where the domestic price level rises and investor welfare falls (Condition 33). These results illustrate the sensitivity of different margins to unilateral trade liberalization.

4.3.1. Bilateral Trade Liberalization

Figure 2 shows that when both countries simultaneously increase trade costs by equal amounts, domestic workers suffer welfare losses identical to those of foreign workers, and domestic investors experience welfare losses identical to those of foreign investors. Thus, bilateral trade liberalization produces symmetric welfare gains for workers and investors in both countries.

Real Income Change under Bilateral Trade Liberalization

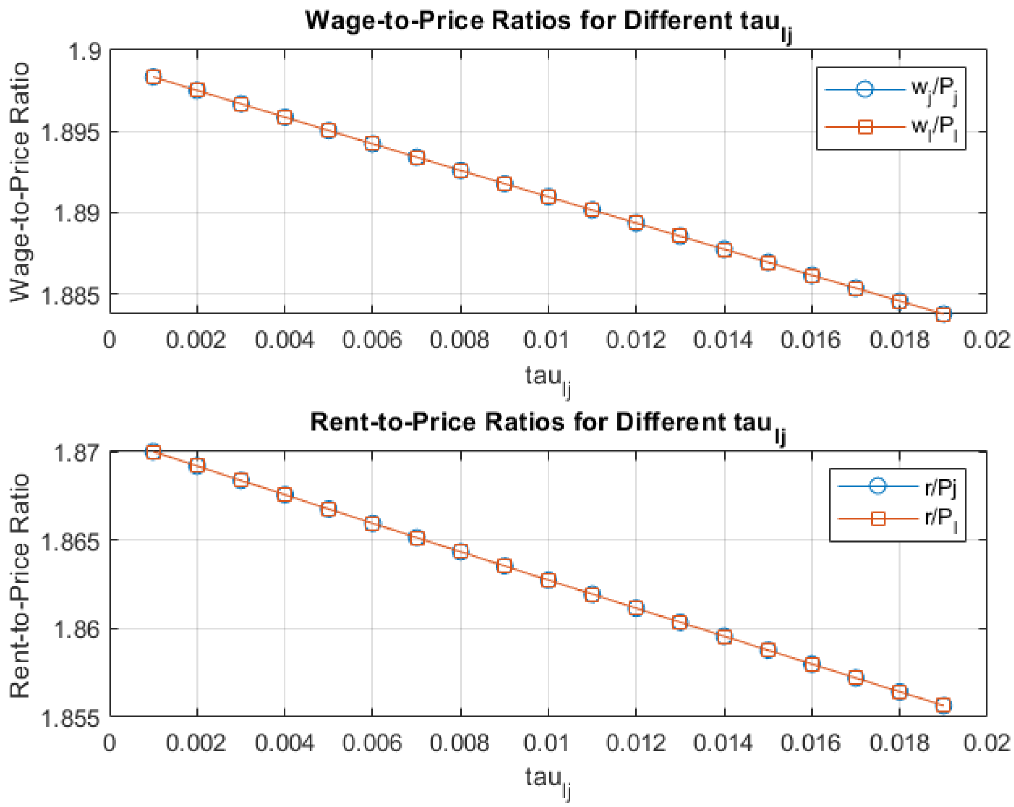

FIGURE 2. Real income change with symmetric change of trade cost

4.3.2. Unilateral Trade Liberalization

Using the reference ranges in Figure 1, I examine four values of ξ to illustrate how welfare outcomes shift as labor intensity changes. The selected values represent different regimes:

- $\xi = 0.75 \in [0.697, 1]$
- $\xi = 0.67 \in [0.096, 0.697)$
- $\xi = 0.09 \in [0.025, 0.096)$
- $\xi = 0.02 \in [0, 0.025)$

Case 1: $\xi = 0.75$. Figure 3 shows that both workers and investors in both countries lose from a unilateral tariff increase, i.e, both of agents gain from unilateral trade liberalization. At the same time, foreign workers gain more than domestic workers, and domestic investors benefit more than their foreign counterparts. Since the rental price r is the numéraire, the welfare gains for investors imply a decline in the domestic price index.

Correspondingly, Figure 4 shows that the export share of GDP increases with liberalization.

Real Income Change under Unilateral Tariff Reduction (ξ = 0.75)

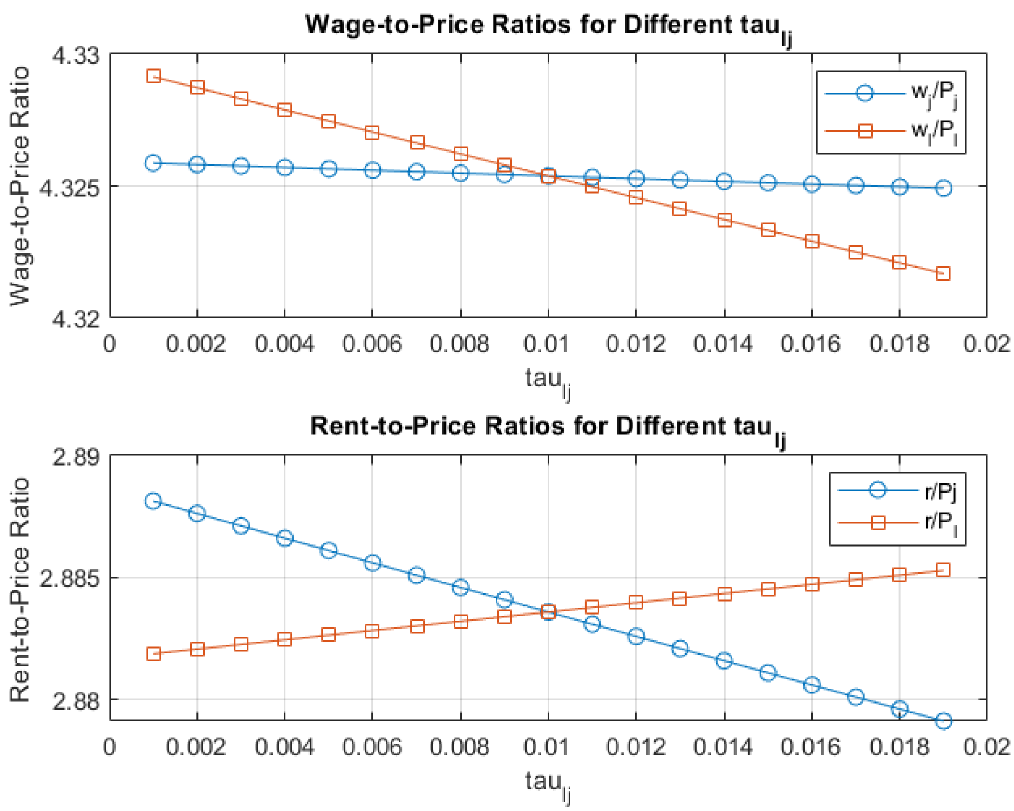

FIGURE 3. Real income change with unilateral tariff reduction, ξ = 0.75.

Export Share Response under Unilateral Tariff Reduction
(ξ = 0.75)

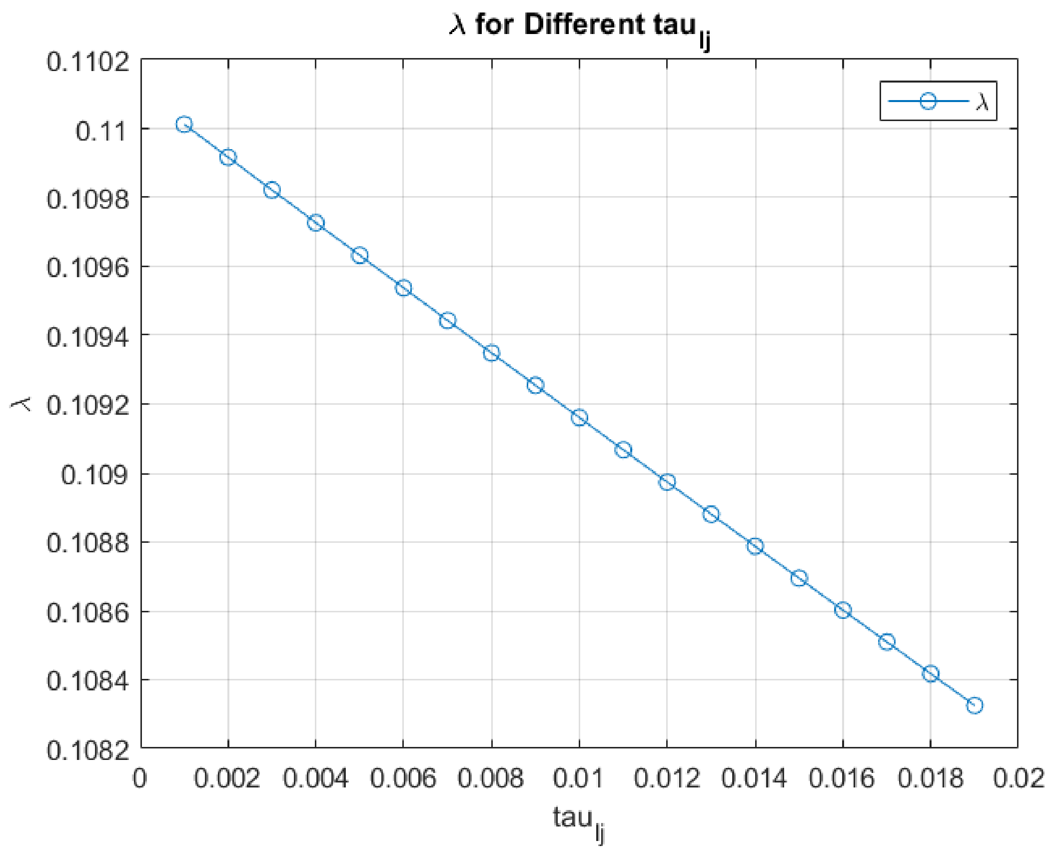

FIGURE 4. Export share response with unilateral tariff change, $\xi = 0.75$.

Case 2: $\xi = 0.67$. When $\xi = 0.67$—a value consistent with empirical labor intensity based on national accounts—domestic workers lose from a tariff decrease, as shown in Figure 5. Although the domestic price index falls with lower tariffs, allowing domestic investors to benefit, the price reduction is not sufficient to offset the decline in wages, resulting in a net welfare loss for workers. The export share of GDP declines in response to the unilateral tariff increase, as expected, since Condition 39 is not satisfied (see Figure 6).

Real Income Change under Unilateral Tariff Change ($\xi = 0.67$)

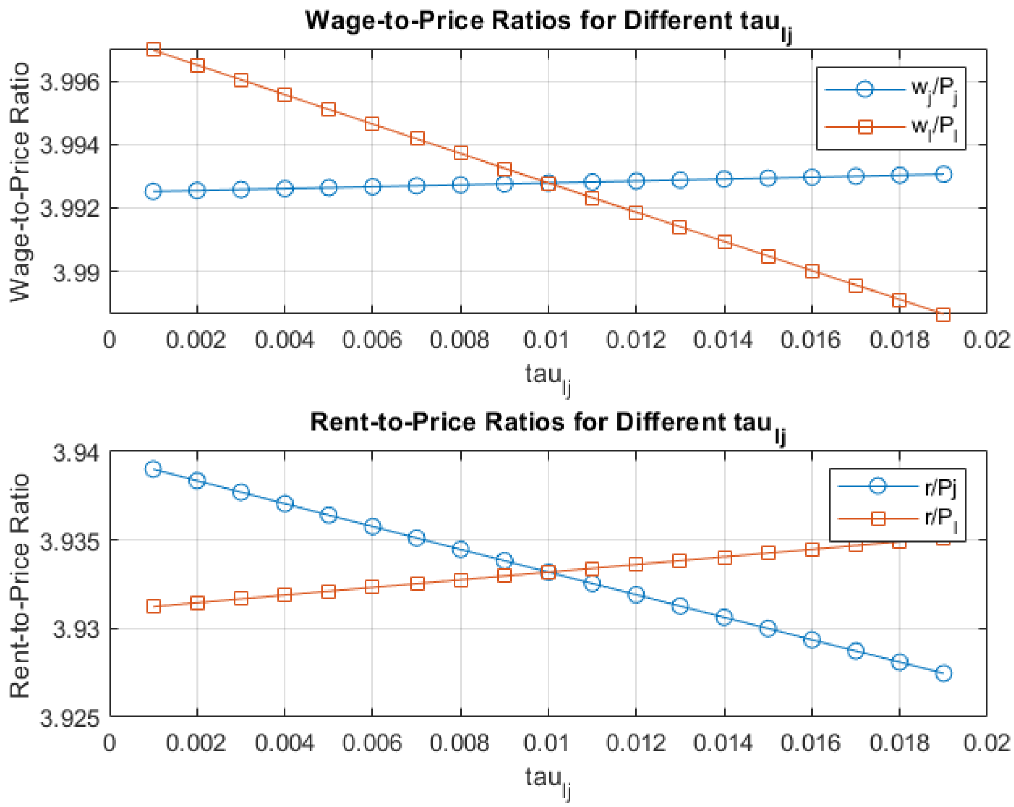

FIGURE 5. Real income change with unilateral tariff change, $\xi = 0.67$.

Export Share Response under Unilateral Tariff Change (ξ = 0.67)

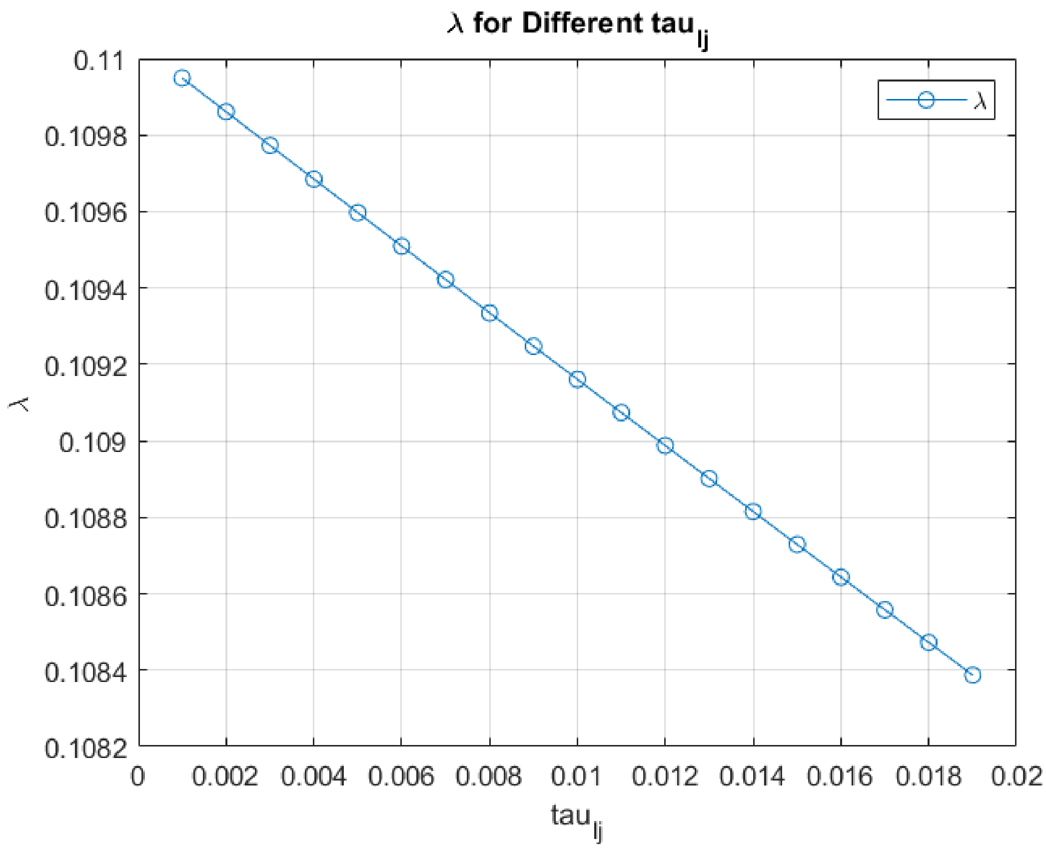

FIGURE 6. Export share response with unilateral tariff change, ξ = 0.67.

Case 3: ξ = 0.09. With ξ = 0.09, the welfare patterns remain similar to the ξ = 0.67 case, as shown in Figure 7. However, the export share of GDP now increases in response to a unilateral tariff increase, as illustrated in Figure 8.

Real Income Change under Unilateral Tariff Change (ξ = 0.09)

FIGURE 7. Real income change with unilateral tariff change, ξ = 0.09.

Export Share Response under Unilateral Tariff Change (ξ = 0.09)

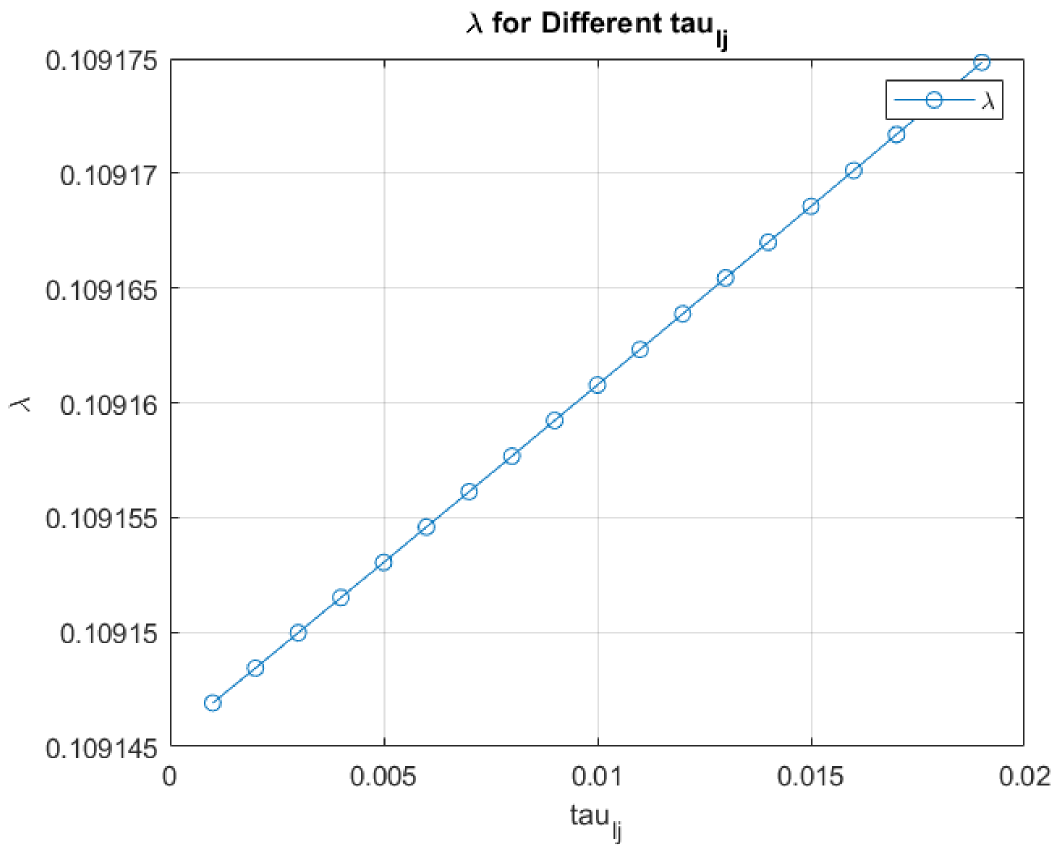

FIGURE 8. Export share response with unilateral tariff change, ξ = 0.09.

Case 4: ξ = 0.02. At ξ = 0.02, domestic investors begin to gain from a unilateral tariff increase, suggesting that the domestic price index now decreases with rising tariffs (Figure 9). Export share also increases with tariff increase, as shown in Figure 10.

Real Income Change under Unilateral Tariff Change (ξ = 0.02)

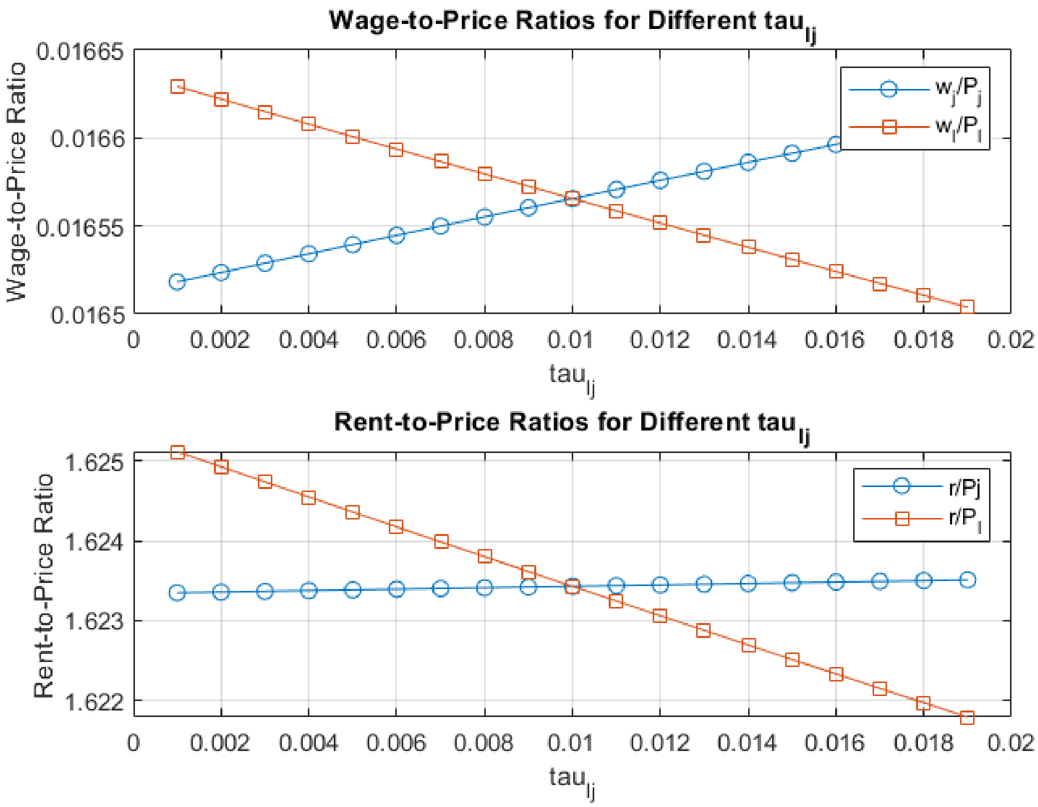

FIGURE 9. Real income change with unilateral tariff change, ξ = 0.02.

Export Share Response under Unilateral Tariff Change (ξ = 0.02)

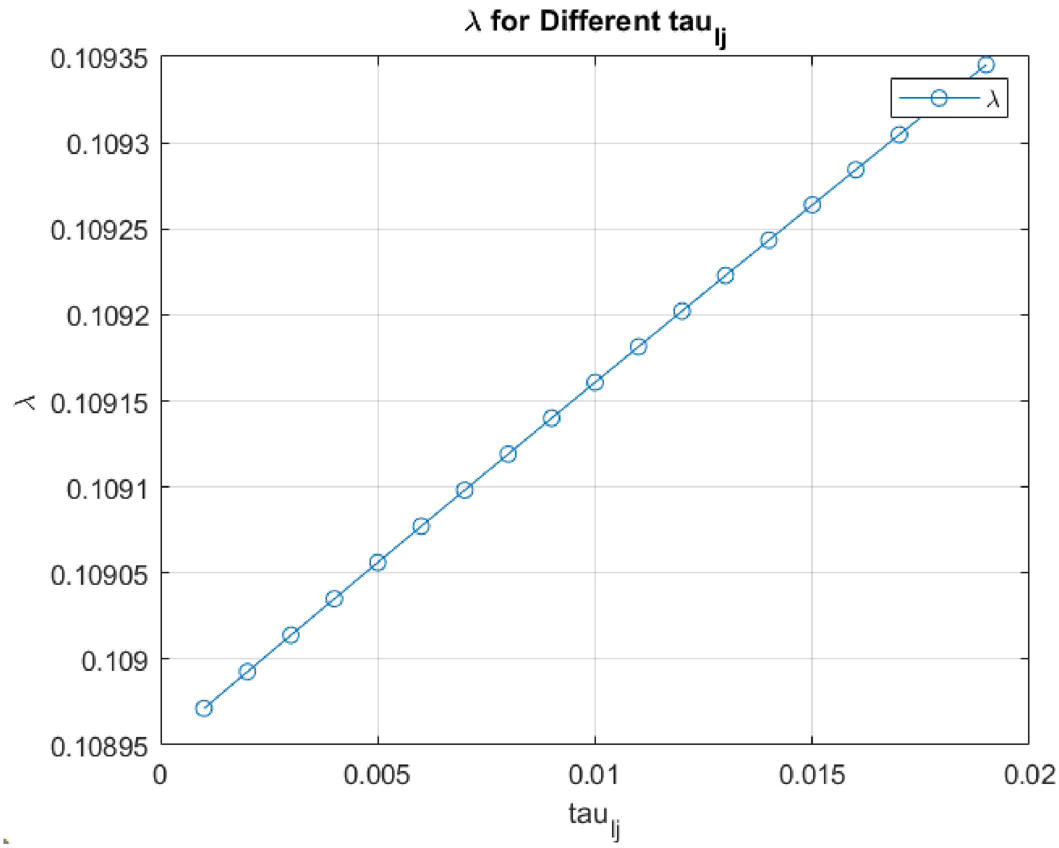

FIGURE 10. Export share response with unilateral tariff change, ξ = 0.02.

4.4. Summary

Introducing a capital market that allows mobility across countries does not alter the welfare implications of bilateral trade liberalization. However, it can significantly influence the welfare outcomes of unilateral trade liberalization, depending on the intensity of capital as an input in production. Typically, studies calibrate $\xi = 0.67$, implying that labor is relatively more important than capital. With this ξ value, my numeric analysis shows that reducing trade barriers yields welfare loss for domestic workers since the negative wage effect can't be offset by the domestic price decrease. As ξ decreases, meaning capital becomes a more critical input for production, trade liberalization can lead to export share decrease and domestic price increase.

This result also suggests that goods trade and capital mobility are complementary when labor remains immobile, as illustrated in the figures in Appendix E. In other words, a unilateral increase in goods trade induces capital outflows. When capital intensity is high, such liberalization can lead to welfare losses.

5. Conclusion

I have shown that allowing one production factor to be mobile across two countries fundamentally alters the welfare implications of unilateral trade liberalization, particularly for workers. In the absence of factor mobility, unilateral trade liberalization always leads to welfare gains for all agents, as demonstrated by Arkolakis, Costinot, and Rodríguez-Clare (2012) and Appendix F. The key mechanism introduced by factor mobility is its impact on firm entry (or the mass of firms).

In the Arkolakis, Costinot, and Rodríguez-Clare (2012) framework, where production factors are immobile, an increase in trade does not influence the mass of domestic firms but instead affects only the price index. Specifically, in the Melitz (2003) setup, the domestic mass of firms is solely determined by the domestic population. Since the population remains unchanged, the number of domestic firms remains fixed. As a result, international trade preserves domestic production while increasing the average productivity of surviving firms. Import competition in this setting occurs only among domestic firms, and the total global market share of firms from any given country remains unchanged. For example, if country J accounts for 50% of the global market share before trade liberalization, making J to be more open to trade will not expand or shrink its total market share; rather, it will simply increase the average productivity of its domestic firms.

However, when capital is mobile across borders, the mass of firms becomes a function of both population and wages, with the rental price serving as the numeraire. This contrasts with Arkolakis, Costinot, and Rodríguez-Clare (2012)-type models, where the domestic wage can be set as the numeraire and thus remains unchanged. Under unilateral trade liberalization, increased domestic demand for foreign goods induces capital outflows, which reduces the mass of domestic firms.

The negative effect of a decline in the mass of firms may be fully offset by a reduction in production costs. When ξ is sufficiently large, the intensive margin effect dominates the extensive margin effect, allowing domestic production to be sustained and resulting in welfare gains for both workers and investors. In contrast, when production is capital-intensive, the extensive margin effect dominates: too few firms survive, domestic production contracts sharply, and both types of domestic agents ultimately suffer welfare losses. This result highlights how capital mobility can overturn standard welfare predictions by relaxing the implicit fixed-entry condition embedded in the Arkolakis, Costinot, and Rodríguez-Clare (2012) framework.

References

Acemoglu, Daron, David Autor, David Dorn, Gordon H. Hanson, and Brendan Price. 2016. "Import Competition and the Great US Employment Sag of the 2000s." *Journal of Labor Economics* 34 (S1): S141–S198. https://doi.org/10.1086/682384.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review* 102 (1): 94–130. https://www.aeaweb.org/articles?id=10.1257/aer.102.1.94.

Armington, Paul S. 1969. "A Theory of Demand for Products Distinguished by Place of Production (Une théorie de la demande de produits différenciés d'après leur origine) (Una teoría de la demanda de productos distinguiéndolos según el lugar de producción)." *Staff Papers (International Monetary Fund)* 16 (1): 159–178. http://www.jstor.org/stable/3866403.

Autor, David, David Dorn, Gordon H. Hanson, Gary Pisano, and Pian Shu. 2020. "Foreign Competition and Domestic Innovation: Evidence from US Patents." *American Economic Review: Insights* 2 (3): 357–74. https://www.aeaweb.org/articles?id=10.1257/aeri.20180481.

Autor, David H., David Dorn, and Gordon H. Hanson. 2013a. "The China Syndrome: Local Labor Market Effects of Import Competition in the United States." *American Economic Review* 103 (6): 2121–68. https://www.aeaweb.org/articles?id=10.1257/aer.103.6.2121.

Autor, David H., David Dorn, and Gordon H. Hanson. 2013b. "The China Syndrome: Local Labor Market Effects of Import Competition in the United States." *American Economic Review* 103 (6): 2121–68. https://www.aeaweb.org/articles?id=10.1257/aer.103.6.2121.

Bai, Yan, Keyu Jin, and Dan Lu. 2024. "Misallocation under Trade Liberalization." *American Economic Review* 114 (7): 1949–85. https://www.aeaweb.org/articles?id=10.1257/aer.20200596.

Boulhol, Hervé. 2009. "Do capital market and trade liberalization trigger labor market deregulation?" *Journal of International Economics* 77 (2): 223–233. https://www.sciencedirect.com/science/article/pii/S0022199609000075.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro. 2019. "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock." *Econometrica* 87 (3): 741–835. https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA13758.

Caliendo, Lorenzo, Robert C Feenstra, John Romalis, and Alan M Taylor. 2021. "A Second-best Argument for Low Optimal Tariffs." Working Paper 28380, National Bureau of Economic Research. http://www.nber.org/papers/w28380.

Chaney, Thomas. 2008. "Distorted Gravity: The Intensive and Extensive Margins of International Trade." *The American Economic Review* 98 (4): 1707–1721. http://www.jstor.org/stable/29730143.

Costinot, Arnaud, Guido Lorenzoni, and Iván Werning. 2014. "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation." *Journal of Political Economy* 122 (1): 77–128. https://doi.org/10.1086/674425.

Eaton, Jonathan, and Samuel Kortum. 2002. "Technology, Geography, and Trade." *Econometrica* 70 (5): 1741–1779. http://www.jstor.org/stable/3082019.

Edwards, Sebastian, and Sweder van Wijnbergen. 1986. "The Welfare Effects of Trade and Capital Market Liberalization." *International Economic Review* 27 (1): 141–148. http://www.jstor.

org/stable/2526611.

Foley, C. Fritz, and Kalina Manova. 2014. "International Trade, Multinational Activity, and Corporate Finance." Working Paper 20634, National Bureau of Economic Research. http://www.nber.org/papers/w20634.

Harrison, Ann, John McLaren, and Margaret McMillan. 2011. "Recent Perspectives on Trade and Inequality." *Annual Review of Economics* 3 (Volume 3, 2011): 261–289. https://www.annualreviews.org/content/journals/10.1146/annurev.economics.102308.124451.

Hu, Chenyue. 2024. "Portfolio Choice Analysis in a Multi-country Macro Model."

Humphrey, Thomas M. 1987. "Classical and Neoclassical Roots of the Theory of Optimum Tariffs." *FRB Richmond Economic Review* 73 (4): 17–28. https://ssrn.com/abstract=2125244.

Jones, Ronald W. 1967. "International capital movements and the theory of tariffs and trade." *The Quarterly Journal of Economics* 81 (1): 1–38.

Kemp, Murray C. 1966. "The gain from international trade and investment: A neo-Heckscher-Ohlin approach." *The American Economic Review* 56 (4): 788–809.

Kleinman, Benny, Ernest Liu, Stephen J Redding, and Motohiro Yogo. 2023. "Neoclassical Growth in an Interdependent World." Working Paper 31951, National Bureau of Economic Research. http://www.nber.org/papers/w31951.

Krugman, Paul. 1991. "Increasing Returns and Economic Geography." *Journal of Political Economy* 99 (3): 483–499. https://doi.org/10.1086/261763.

Lloyd, Simon P., and Emile A. Marin. 2024. "Capital controls and trade policy." *Journal of International Economics* 151: 103965. https://www.sciencedirect.com/science/article/pii/S0022199624000928.

Markusen, James R., and Lars E. O. Svensson. 1985. "Trade in Goods and Factors with International Differences in Technology." *International Economic Review* 26 (1): 175–192. http://www.jstor.org/stable/2526533.

Martin, Philippe, and Carol Ann Rogers. 1995. "Industrial location and public infrastructure." *Journal of International Economics* 39 (3): 335–351. https://www.sciencedirect.com/science/article/pii/0022199695013766.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71 (6): 1695–1725. https://onlinelibrary.wiley.com/doi/abs/10.1111/1468-0262.00467.

Melitz, Marc J., and Gianmarco I. P. Ottaviano. 2008. "Market Size, Trade, and Productivity." *The Review of Economic Studies* 75 (1): 295–316. https://doi.org/10.1111/j.1467-937X.2007.00463.x.

Mundell, Robert A. 1957. "International Trade and Factor Mobility." *The American Economic Review* 47 (3): 321–335. http://www.jstor.org/stable/1811242.

Purvis, Douglas D. 1972. "More on growth and the balance of payments: The adjustment process." *The Canadian Journal of Economics/Revue canadienne d'Economique* 5 (4): 531–540.

Rodrik, Dani. 2016. "Premature deindustrialization." *Journal of economic growth* 21: 1–33.

Svensson, Lars EO. 1984. "Factor trade and goods trade." *Journal of International Economics* 16 (3-4): 365–378.

Wong, Kar-yiu. 1986. "c." *Journal of International Economics* 21 (1): 25–43. https://www.

sciencedirect.com/science/article/pii/0022199686900036.

Appendix A. Matrix Form with $\hat{N}_J$ and $\hat{N}_L$

$$\begin{pmatrix} 1 & 0 & \xi-1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \xi-1 & 0 & 0 \\ \sum_I V_{JI} & 0 & (1-\xi(1-\epsilon))\sum_I V_{JI} & 0 & \frac{\alpha}{1-\sigma}V_{JJ} & \frac{\alpha}{1-\sigma}V_{JL} \\ 0 & \sum_I V_{LI} & 0 & (1-\xi(1-\epsilon))\sum_I V_{LI} & \frac{\alpha}{1-\sigma}V_{LJ} & \frac{\alpha}{1-\sigma}V_{LL} \\ 0 & 0 & \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\ -V_{JJ} & -V_{LJ} & \xi\sum_I V_{JI}-\xi(1-\epsilon)V_{JJ} & -\xi(1-\epsilon)V_{LJ} & \frac{\alpha}{1-\sigma}(V_{JJ}+V_{LJ}) & 0 \end{pmatrix} \begin{pmatrix} \hat{N}_J \\ \hat{N}_L \\ \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -\alpha V_{JL}\hat{\tau}_{JL} \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \end{pmatrix}$$

Appendix B. Term $A > 0$

First, we evaluate the sign of A. It is reasonable to assume that the fixed cost for exporting is bigger than fixed cost for selling in the domestic market, and the iceberg cost will never be negative. Given these assumption, I have $\lambda < \frac{1}{2}$, i.e, the proportion of GDP from exporting is always less than 50%. Then I have $1-\lambda > \frac{1}{2}$. So $A > (1-\xi) + 2\lambda\xi + 2\lambda(\epsilon\xi - 1) = (1-\xi) + 2\lambda\xi + 2\lambda\epsilon\xi - 2\lambda = (1-\xi) + 2\lambda(\xi-1) + 2\lambda\epsilon\xi = (1-\xi)(1-2\lambda) + 2\lambda\epsilon\xi > 0$.

Appendix C. Order of conditions

I will list these conditions again here:

First the condition for price index to increase with a unilateral trade liberalization, thus make investors lose welfare from the unilateral trade liberalization:

$$\lambda((1-2\lambda)(1-\epsilon\xi)-\xi) > \epsilon\xi \tag{A1}$$

Then the condition for workers to lose welfare from the unilateral trade liberalization:

$$(1+\frac{\xi}{\sigma-1})\alpha(1-\lambda)+\lambda((1-2\lambda)(1-\epsilon\xi)-\xi)>\epsilon\xi \tag{A2}$$

Finally the condition for export share of GDP to decrease with a unilateral trade liberalization:

$$(1-2\lambda(1-\epsilon\xi))-\xi>\epsilon\xi \tag{A3}$$

Given the common term one can see immediately that Condition A1 is stricter than Condition A2. Additionally, assume $\lambda<\frac{1}{2}$, it is also immediate that Condition A1 is stricter than Condition A3.

To decide the strictness between Condition A3 and Condition A2, I firstly assume that Condition A3 is satisfied, then I will prove that Condition A2 is automatically satisfied after.

Assume Condition A3 is satisfied gives:

$$\lambda((1-2\lambda)(1-\epsilon\xi)-\xi)>\lambda\epsilon\xi$$

Substitute the above term back to Condition A2, I get:

$$(1+\frac{\xi}{\sigma-1})\alpha(1-\lambda)+\lambda((1-2\lambda)(1-\epsilon\xi)-\xi)>(1+\frac{\xi}{\sigma-1})\alpha(1-\lambda)+\lambda\epsilon\xi$$

If I can prove that

$$(1+\frac{\xi}{\sigma-1})\alpha(1-\lambda)+\lambda\epsilon\xi>\epsilon\xi \tag{A4}$$

Then I am done. To prove Condition A4 is true, I spread the terms including spreading ϵ as $\frac{\sigma\alpha}{\sigma-1}$ and I have:

$$(1+\frac{\xi}{\sigma-1})(\alpha-\alpha\lambda)+\lambda\frac{\sigma\alpha}{\sigma-1}\xi>\frac{\sigma\alpha}{\sigma-1}\xi$$

This is the same as proving:

$$\alpha-\alpha\lambda+\frac{\alpha\xi}{\sigma-1}-\frac{\alpha\lambda\xi}{\sigma-1}+\lambda\frac{\sigma\alpha}{\sigma-1}\xi-\frac{\sigma\alpha}{\sigma-1}\xi>0$$

Re-arrange the terms gives:

$$\alpha(1-\lambda)+\frac{\alpha\xi}{\sigma-1}(1-\lambda+\lambda\sigma-\sigma)>0$$

Re-arrange the terms further gives:

$$
\begin{aligned}
&\alpha(1-\lambda)+\frac{\alpha\xi}{\sigma-1}((1-\lambda)+(\lambda-1)\sigma)\\
&=\alpha(1-\lambda)+\frac{\alpha\xi}{\sigma-1}(1-\lambda)(1-\sigma)\\
&=\alpha(1-\lambda)-\alpha\xi(1-\lambda)\\
&=\alpha(1-\lambda)(1-\xi)>0
\end{aligned}
\tag{A5}
$$

So I proved that Condition A3 satisfied meaning Condition A2 automatically get satisfied, which means I can order these three conditions in there strictness as: Condition A2 is the loosest, Condition A3 is in the middle, and Condition A1 is the strictest.

Appendix D. Proof for symmetric equilibrium solution

In this section I will calculate the solution in the case where $\gamma = \frac{1}{2}$ and

$$F_J^E = F_L^E = F^E$$

$$F_{JJ} = F_{LL} = F_0$$

$$F_{JL} = F_{LJ} = F_1$$

$$L_J = L_L = L$$

$$\tau_{JL} = \tau_{LJ} = \tau$$

$$\tau_{JJ} = \tau_{LL} = 0$$

I can re-write the equilibrium conditions as follows:

$$\sum_I V_{JI} = \frac{\sigma\alpha}{\sigma-1} N_J F^E W_J^{\xi} r^{1-\xi}$$

$$\sum_I V_{LI} = \frac{\sigma\alpha}{\sigma-1} N_L F^E W_L^{\xi} r^{1-\xi}$$

$$\sum_I V_{JI} = \frac{W_J L}{\xi}$$

$$\sum_I V_{LI} = \frac{W_L L}{\xi}$$

$$rK_0 = (1-\xi)(\sum_I V_{JI} + \sum_I V_{LI})$$

$$W_J L + \gamma r K_0 = V_{JJ} + V_{LJ}$$

Also, for V_{JI} specifically, I have:

$$V_{JI} = \frac{\sigma\alpha\underline{z}^{\alpha}}{\alpha+1-\sigma} N_J B_I^{\frac{\alpha}{\sigma-1}} F_{JI}^{1+\frac{\alpha}{1-\sigma}} (W_J^{\xi} r^{1-\xi})^{1-\epsilon}(1+\tau_{JI})^{-\alpha}$$

So I can get:

$$\frac{V_{JJ}}{V_{LL}} = \frac{N_J}{N_L}(\frac{B_J}{B_L})^{\frac{\alpha}{\sigma-1}}(\frac{W_J}{W_L})^{\xi(1-\epsilon)} = f$$
$$\frac{V_{JL}}{V_{LJ}} = \frac{N_J}{N_L}(\frac{B_L}{B_J})^{\frac{\alpha}{\sigma-1}}(\frac{W_J}{W_L})^{\xi(1-\epsilon)} = g$$

Note $g = f + \delta$, if $\delta = 0$, it means $f = g$, if $\delta > 0$, $f < g$, and if $\delta < 0$, it means $f > g$. Using this notation, I have $\frac{\sum_I V_{JI}}{\sum_I V_{LI}} = \frac{fV_{LL}+gV_{LJ}}{V_{LL}+V_{LJ}} = f + \tilde{\delta}$ where $\tilde{\delta}$ works similarly as δ. The left hand side of trade balance condition for country J thus becomes:

$$W_J L + \gamma r K_0 = \xi \sum_I V_{JI} + \frac{1-\xi}{2}(\sum_I V_{JI} + \sum_I V_{LI}) = (\xi + (1-\xi)\frac{f+\tilde{\delta}+1}{2(f+\tilde{\delta})})\sum_I V_{JI}$$

The right hand side is:

$$V_{JJ} + \frac{1}{f}V_{JL}$$

The trade balance condition now becomes:

$$(\xi + (1-\xi)\frac{f+\tilde{\delta}+1}{2(f+\tilde{\delta})})\sum_I V_{JI} = V_{JJ} + \frac{1}{f}V_{JL} \tag{A6}$$

which can be arranged as:

$$((\xi + (1-\xi)\frac{f+\tilde{\delta}+1}{2(f+\tilde{\delta})}) - 1)V_{JJ} + ((\xi + (1-\xi)\frac{f+\tilde{\delta}+1}{2(f+\tilde{\delta})}) - \frac{1}{f})V_{JL} = 0 \tag{A7}$$

Given that $V_{JJ} > 0$ and $V_{JL} > 0$, it must be true that:

$$(\xi + (1-\xi)\frac{f+\tilde{\delta}+1}{2(f+\tilde{\delta})}) - 1 = (\xi + (1-\xi)\frac{f+\tilde{\delta}+1}{2(f+\tilde{\delta})}) - \frac{1}{f} = 0$$

which means: $f = 1$ and $\tilde{\delta} = 0$. From there, one can easily get that $B_J = B_L$, $W_J = W_L$, $N_J = N_L$, and thus $V_{JJ} = V_{JL}$, $V_{JL} = V_{LJ}$

Appendix E. Commodity movement and factor movement is complement

Given $\xi = 0.67$, the amount of capital that got invested in domestic production is increasing with the increasing of the trade cost, i.e, capital will outflow to the foreign country when having a unilateral trade liberalization. At the same time, the mass of firms is having the same trend.

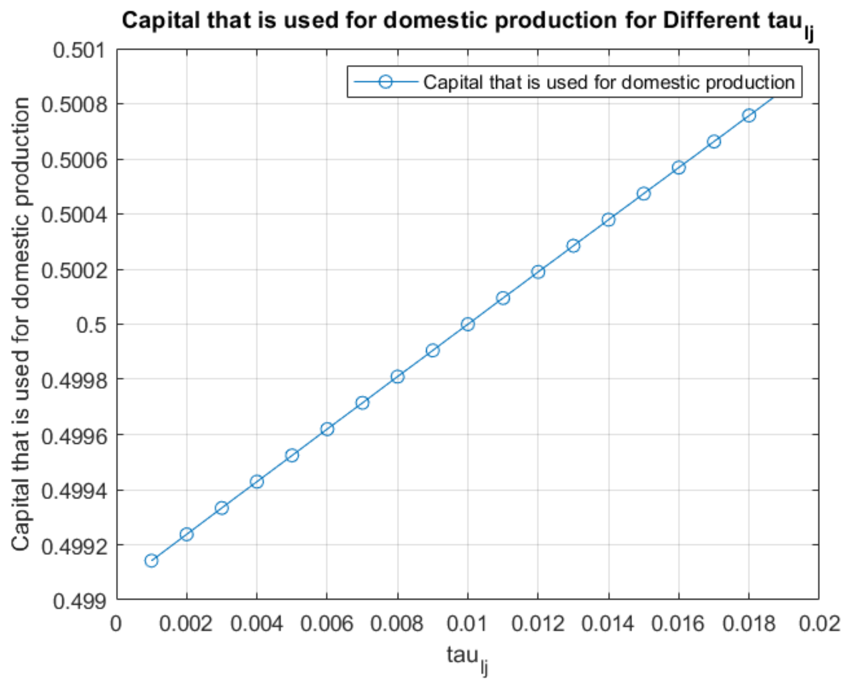

FIGURE A1. Change of capital that is used in domestic production with the change of τ_{LJ}

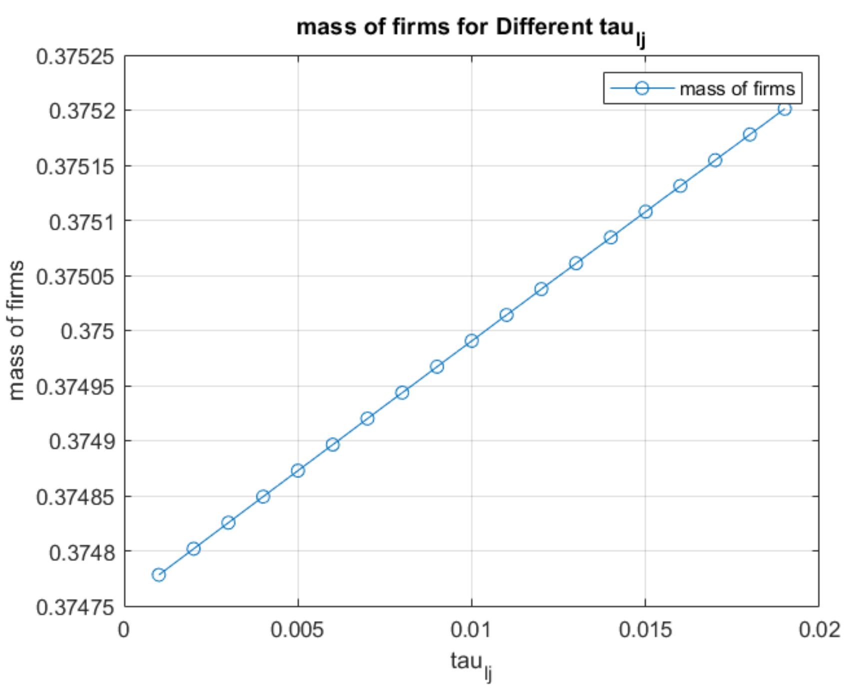

FIGURE A2. Change of the mass of firms with the change of τ_{LJ}

Appendix F. No capital market (Melitz Chaney setup)

In this section, I will let $\xi = 1$ to show the effect of bilateral and unilateral trade liberalization in the original Melitz (2003) model, where both the bilateral and unilateral trade liberalization is bringing welfare gains. The equilibrium conditions become as follows:

First, free entry conditions or the zero profit conditions will nail down the mass of firms in each country:

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{JI} = N_J F_J^E W_J$$

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{LI} = N_L F_L^E W_L$$

Then the labor clearing conditions give:

$$L_J = \frac{\sum_I V_{JI}}{W_J}$$

$$L_L = \frac{\sum_I V_{LI}}{W_L}$$

Finally, trade balance conditions ensure the total expenditure equals to the total cost:

$$W_J L_J = V_{JJ} + V_{LJ}$$

$$W_L L_L = V_{JL} + V_{LL}$$

Combine the free entry conditions and labor market clearing conditions gives:

$$L_J = \frac{\sigma-1}{\sigma\alpha} N_J F_J^E$$

$$L_L = \frac{\sigma-1}{\sigma\alpha} N_L F_L^E$$

The solution for the symmetric case is easy to get which is $N_J = N_L$, $B_J = B_L$, and $W_J = W_L$.

Note $\frac{\sigma\alpha}{\sigma-1} = \epsilon$, and log differentiate the equilibrium conditions with only the change of $\hat{\tau}_{SI}$, and define W_J as the numeraire I have:

$$\begin{pmatrix} \epsilon & -\frac{\alpha}{\sigma-1}V_1 & -\frac{\alpha}{\sigma-1}V_2 \\ \epsilon-1 & -\frac{\alpha}{\sigma-1} & +\frac{\alpha}{\sigma-1} \\ 0 & -\frac{\alpha}{\sigma-1}V_1 & -\frac{\alpha}{\sigma-1}V_2 \end{pmatrix} \begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha V_2 \hat{\tau}_{LJ} \\ \alpha \hat{\tau}_{JL} - \alpha \hat{\tau}_{LJ} \\ -\alpha V_2 \hat{\tau}_{JL} \end{pmatrix}$$

where $V_1 = \frac{V_{JJ}}{V_{JJ}+V_{JL}} = \frac{V_{LL}}{V_{LL}+V_{LJ}}$ and $V_2 = 1 - V_1$

F.1. Bilateral trade liberalization

When we assume $\hat{\tau}_{JL} = \hat{\tau}_{LJ} = \hat{\tau} < 0$, we get:

$$\begin{pmatrix} \hat{W}_L = 0 \\ \hat{B}_J = V_2(\sigma - 1)\hat{\tau} \\ \hat{B}_L = V_2(\sigma - 1)\hat{\tau} \end{pmatrix}$$

With $-P_I = -\frac{\hat{B}_I}{\sigma - 1}$, both countries enjoy a welfare gain of $-V_2\hat{\tau}$, where $V_2 = \frac{V_{JL}}{VJL + V_{JJ}}$ so the welfare gains of the bilateral trade liberalization is the same as the case with footloose capital.

F.2. Unilateral trade liberalization

when I have $\hat{\tau}_{LJ} = \hat{\tau} < 0$ and $\hat{\tau}_{JL} = 0$

$$\begin{pmatrix} \hat{W}_L = -\frac{\alpha V_1 \hat{\tau}}{\epsilon + (\epsilon - 1)(V_1 - V_2)} \\ \hat{B}_J = \frac{(\sigma - 1)(\epsilon V_2 V_1 + V_2^2)\hat{\tau}}{\epsilon + (\epsilon - 1)(V_1^2 - V_2^2)} \\ \hat{B}_L = -\frac{(\sigma - 1)(V_2 V_1 + \epsilon V_1^2)}{\epsilon + (\epsilon - 1)(V_1^2 - V_2^2)} \end{pmatrix}$$

Where

$$\hat{W}_J - \hat{P}_J = -\hat{P}_J = -\frac{1}{\sigma - 1}\hat{B}_J = -\frac{(\epsilon V_2 V_1 + V_2^2)\hat{\tau}}{\epsilon + (\epsilon - 1)(V_1 - V_2)} > 0$$

Without the competition for capitals, for country J, decreasing trade barrier is always good, however, with the competition for capitals, from the previous sections, it brings welfare loss.

F.3. Numerical Analysis

Using the fixed cost and Parato distribution parameter as Table 1, a bilateral trade liberalization will give the result as Figure **??** and a unilateral trade liberalization will give the result as Figure ??

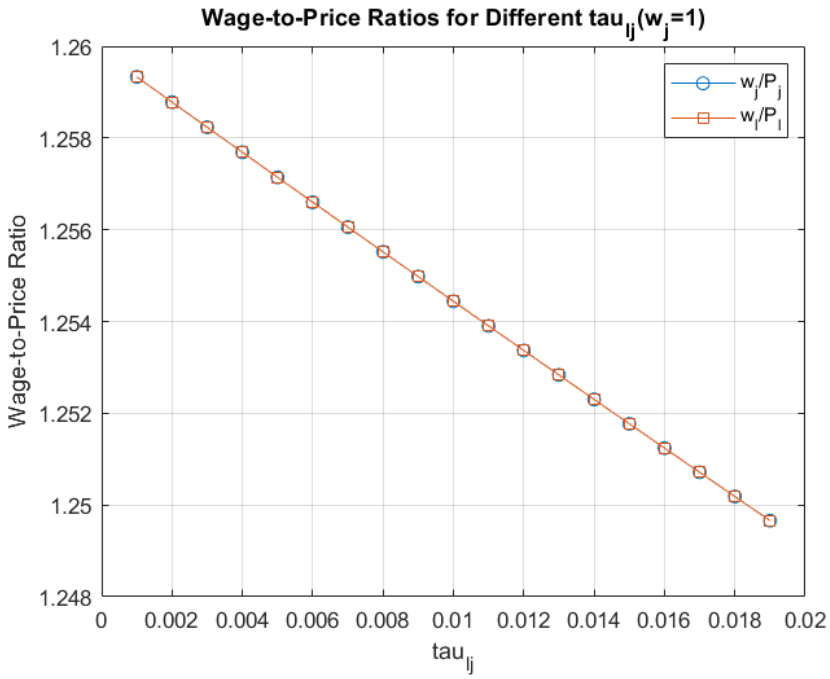

FIGURE A3. Change of real wages with the change of $\tau_{LJ} = \tau_{JL}$

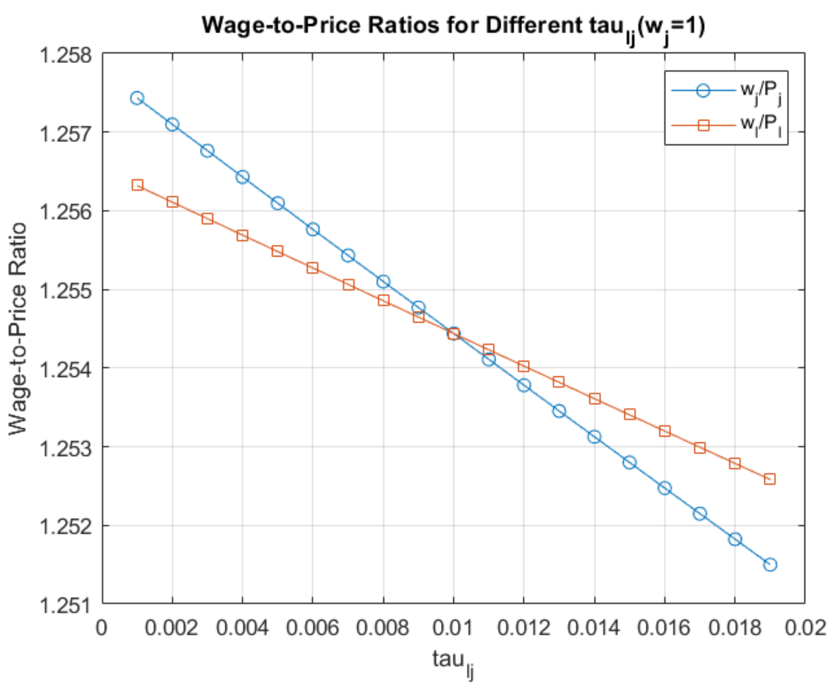

FIGURE A4. Change of real wages with the change of τ_{LJ}