Globalization with Capital Integration and Spatial Entry*

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Abstract

What does it mean when factories ”leave” a country? In standard trade models with heterogeneous firms entry, trade liberalization reallocates productive resources toward more productive domestic firms, leaving aggregate productive capacity unchanged. Yet factory closures are often accompanied by asset sales and new investments abroad, suggesting that productive capacity can relocate across national borders. To study this mechanism, I extend the Melitz (2003) model by introducing an integrated capital market in which capital flows freely across locations while labor remains immobile. Using exact hat algebra and a 2017 region-by-region trade matrix covering the 50 U.S. states, Washington, D.C., and China, I quantify the effects of President Trump’s 2018 tariffs on Chinese imports. The model-predicted percentage change in the number of firms is positively correlated with the change in the number of establishments reported by the County Business Patterns (CBP) across states. Additionally, I find that workers in all U.S. regions experience welfare gains from the tariffs, and these gains are positively correlated with Republican vote share in the 2016 presidential election.

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1 Introduction

"Too many factories where folks thought they'd retire one day have left town. Too many jobs that provided a decent living have been shipped overseas."

— Barack Obama

The idea that factories can "leave" a country is pervasive in public discussions of globalization. But what does it mean for a factory to leave? Physical structures often remain in place, and land itself cannot relocate. Yet a factory is more than a collection of buildings. It is a productive organization that relies on a broad set of productive resources, including equipment, labor, managerial resources, and future investment. When a factory closes in one location and production emerges elsewhere, what has changed is not the location of the building, but the allocation of the productive resources that support production activity.

Consider Whirlpool's closure of its refrigerator plant in Evansville, Indiana, and the relocation of production to Mexico in 2010. The closure was reflected not merely by an empty building, but also in Whirlpool's financial reports through restructuring expenses, proceeds from asset sales, and new investments in production facilities abroad. In this sense, the statement that a factory "left" describes a redirection of investment and production activity from one location to another.

Yet this interpretation differs from the way factory closures are typically understood in standard trade models. For example, in the Melitz (2003) framework, trade liberalization causes less productive firms to exit while more productive firms expand. The productive resources employed by exiting firms are not lost; instead, they are reallocated toward more productive domestic activities. Applied to the Whirlpool example, the closure of the Evansville plant would therefore not imply a reduction in U.S. productive capacity. Yet Whirlpool's observed restructuring suggests a different interpretation. The closure was accompanied by asset sales and new investments in production facilities abroad, pointing to a relocation of productive resources across national borders rather than a reallocation within the domestic economy.

This observation raises a natural question: how do the effects of trade liberalization change when productive resources can move across countries rather than being reallocated solely within national borders? To address this question, I extend the Melitz (2003) model by incorporating an integrated capital market in which capital flows frictionlessly across locations while labor remains geographically immobile. Under this setup, capital relocates across borders in response to differences in the return to capital. As a result, firm entry and exit are no longer tied to local factor endowments but instead respond to equilibrium factor prices. Trade policy therefore affects not only which firms produce, but also where production takes place.

In a symmetric two-country economy (Section 2), I show that a unilateral tariff increase attracts capital inflows into the tariff-imposing country. These inflows stimulate not only production by incumbent firms but also the entry and operation of additional firms. The resulting expansion of domestic productive capacity thus occurs along both the intensive and extensive margins. This extensive-margin response plays a central role in the model's welfare implications. In its absence, capital inflows alone are insufficient to offset the increase in consumer prices generated by the tariff. To demonstrate this point, I introduce capital mobility into an Armington framework (Appendix K) that lacks firm heterogeneity and therefore lacks an extensive margin. Although tariffs continue to attract capital inflows, households experience welfare losses because the adverse price effects dominate the gains from increased production.

To apply the model in a more realistic environment, Section 3 extends the two-country framework to a multi-region setting and employs exact hat algebra to conduct counterfactual analyses of President Trump's 2018 unilateral tariffs on Chinese imports. The analysis uses a region-by-region trade matrix constructed from the Freight Analysis Framework (FAF) and USA Trade Online data. The counterfactual results, reported in Section 3.5, indicate that pure wage earners in every U.S. state experience welfare gains under the tariff policy, regardless of whether China retaliates. At the household level, where households are assumed to receive both labor and capital income, most states experience welfare losses. As discussed in Section 3.6, the cross-state pattern of welfare changes is positively correlated with the results of the 2016 U.S. presidential election.

Positioning in the literature Firstly, this work is related to the literature studying capital flows in open economies. The role of factor mobility under the Heckscher–Ohlin (H–O) framework has been well studied (e.g., Mundell (1957), Wong (1986)). In this setting, trade promotes specialization, and factor mobility helps countries attract the factors they need, thereby increasing domestic total output.

Beyond the classic Heckscher–Ohlin (H–O) framework, several studies incorporate capital in different ways. Costinot et al. (2014) use a two-country, two-good endowment model in which the allocation of consumption itself implies the assignment of "capital." Lloyd and Marin (2024) extend Costinot et al. (2014)'s setup by introducing trade costs. Hu (2024) adopt a home-bias framework in which cross-country capital investment corresponds to government-issued equity in each country.

The role of capital in this paper follows the traditional assumption that capital is used for production and fixed costs of entry. In this respect, the model is closely related to Edwards and van Wijnbergen (1986), Boulhol (2009), and Kleinman et al. (2023), where capital serves a similar function. Edwards and van Wijnbergen (1986) adopts the Dixit–Norman trade policy framework, Boulhol (2009) build on a Krugman-type model, while Kleinman et al. (2023) employs an Armington structure; all assume homogeneous firms.

At the same time, this paper is also related to the literature that studies heterogeneous firm behavior in open-economy frameworks. In particular, Melitz (2003) and Chaney (2008) provide the foundational models on which this paper builds. Venables (1985) and Bagwell and Staiger (2012) study firm delocation effects in a Cournot competition framework. Helpman et al. (2004) and Ramondo and Rodríguez-Clare (2013) focus on firms' decisions to export or engage in foreign direct investment (FDI).

Taken together, the main novelty of this paper is to combine capital mobility with heterogeneous-firm behavior in an open-economy framework. Beyond the modeling framework, this paper is also related to a literature that studies the conditions under which trade liberalization or capital outflows can generate welfare losses.

Caliendo et al. (2021) extend the Melitz model to incorporate roundabout production and show that firm markups raise production costs, implying that optimal tariffs can correct the resulting distortions. Melitz and Ottaviano (2008) construct a model with quadratic utility that generates linear demand and show that unilateral liberalization can induce firm relocation and lead to welfare losses. While their mechanism is related to the one explored here, their model does not involve capital relocation. This paper instead highlights an alternative channel: tariffs can generate welfare gains by attracting capital inflows, which lower the effective cost of firm entry and strengthen both the extensive and intensive margins of production.

Lloyd and Marin (2024) show that capital control policies can generate welfare gains under certain tax and tariff policies. In contrast, this paper studies the welfare consequences of capital mobility in the absence of tariff revenue.

Finally, this paper is related to research examining the welfare consequences of the China shock. Caliendo et al. (2019) show that wage rigidities can generate short-run welfare losses resulting from the China shock. Rather than emphasizing sectoral structural change, this paper focuses simply on how capital mobility interacts with trade policy to affect welfare outcomes.

2 Two-Country Model

To illustrate the key mechanism and underlying intuition, the analysis begins with a two-country, one-sector model. This minimal setup highlights the core economic forces at work.

2.1 Model Setup

Suppose there are two countries in the world: country J and country L. The total capital endowment in the world is denoted by K_0. Capital ownership is fixed and determined by the exogenous parameter γ: households in country J own a fraction $\gamma \in (0, 1)$ of the world capital stock, while households in country L own the remaining fraction $1 - \gamma$. Although capital ownership is fixed, households can invest capital in any location without friction, implying perfect capital mobility and a globally equalized rental rate of capital, denoted by r.

2.2 Consumer Sides

Households in both countries share an identical utility function and therefore solve analogous utility-maximization problems. In country $I \in \{J, L\}$, the representative household chooses consumption $q_I(\omega)$ for each variety $\omega \in \Omega_I$, where Ω_I denotes the set of varieties available in country I, to maximize

$$\max_{\{q_I(\omega)\}} U_I = \left(\int_{\omega \in \Omega_I} q_I(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

subject to the budget constraint

$$\int_{\omega \in \Omega_I} p_I(\omega) \, q_I(\omega) \, d\omega = Y_I,$$

where income in country I is given by

$$Y_I = W_I L_I + r K_I.$$

Utility maximization yields the standard CES demand for each variety ω:

$$q_I(\omega) = \frac{p_I(\omega)^{-\sigma}}{P_I^{1-\sigma}} \, Y_I,$$

where P_I denotes the ideal price index in country I, defined as

$$P_I = \left(\int_{\omega \in \Omega_I} p_I(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}}.$$

2.3 Firm Sides

Each firm produces a differentiated variety ω using a Cobb–Douglas production function form given by

$$q(\omega) = z \left(\frac{l}{\xi} \right)^{\xi} \left(\frac{k}{1-\xi} \right)^{1-\xi},$$

where z denotes firm-level productivity. Firms draw their productivity level z from a country-specific distribution prior to production. After observing z, each firm decides whether to operate, export, or exit the market.

Productivity in country $I \in \{J, L\}$ follows a Pareto distribution with shape parameter $\alpha_I > \sigma - 1$ and lower bound $\underline{z}^I$, such that

$$\Pr(Z \geq z) = \mathcal{F}^I(z) = \begin{cases} \left(\dfrac{\underline{z}^I}{z} \right)^{\alpha_I}, & z \geq \underline{z}^I, \\ 1, & z < \underline{z}^I. \end{cases}$$

To keep the model as simple as possible, I assume that productivity follows the same Pareto distribution in both countries, so that $\alpha_J = \alpha_L = \alpha$ and $\underline{z}_J = \underline{z}_L = \underline{z}$.

Trade between countries is subject to a source–destination–specific iceberg trade cost, which do not generate tariff revenue, denoted by $\tau_{SI} \geq 0$, so that the consumer price of a variety produced in country S and sold in country I exceeds its producer price by the constant factor $(1 + \tau_{SI})$. Since firm productivity z is drawn from a continuous distribution, there is a one-to-one mapping between varieties ω and productivity draws z. Introducing the source-country index S to account for iceberg trade costs does not alter this correspondence. I therefore replace the variety-level notation $p_I(\omega)$ and $q_I(\omega)$ with $p_{SI}(z)$ and $q_{SI}(z)$, respectively.

Given the demand $q_{SI}(z)$ faced in market I, a firm of index (S,z) chooses capital and labor to minimize its production cost. Because serving market I requires shipping $(1+\tau_{SI})$ units for each unit consumed, the firm must produce $q_{SI}(z)(1+\tau_{SI})$ units of output.

Cost minimization implies that the amounts of capital and labor required to serve market I are

$$K_{SI}(z) = \frac{1-\xi}{z}\left(\frac{W_S}{r}\right)^{\xi} q_{SI}(z)(1+\tau_{SI}),$$

$$L_{SI}(z) = \frac{\xi}{z}\left(\frac{W_S}{r}\right)^{\xi-1} q_{SI}(z)(1+\tau_{SI}).$$

Under monopolistic competition, firms set prices as a constant markup over marginal cost. Accordingly, the optimal price charged by a firm with productivity z located in country S and selling to country I is

$$p_{SI}(z) = \frac{\sigma}{\sigma-1} W_S^{\xi} r^{1-\xi} \frac{(1+\tau_{SI})}{z}.$$

2.4 Aggregate Values

The value of trade for a variety produced by a firm with productivity z located in country S and sold in country I is given by

$$V_{SI}(z) = p_{SI}(z)\, q_{SI}(z) = \sigma B_I (W_S^{\xi} r^{1-\xi} \frac{1+\tau_{SI}}{z})^{1-\sigma}, \tag{1}$$

where

$$B_I = \frac{W_I L_I + r K_I}{P_I^{1-\sigma}} \left(\frac{1}{\sigma}\right)^{\sigma} \left(\frac{1}{\sigma-1}\right)^{1-\sigma} \tag{2}$$

I interpret B_I as a measure of a *market size*. It increases with aggregate income and with the price index. Higher income raises demand for each variety, while a higher price index reflects weaker competition across varieties, which also increases the demand faced by individual firms. As a result, a larger measure of purchase size corresponds to a more favorable market environment for firm survival.

2.5 Technology Threshold for Selling in a Market

A firm located in country S sells to market I only if its variable profits from serving that market are at least as large as the fixed cost of selling there. Formally, a firm with productivity z must satisfy

$$\pi_{SI}(z) \geq F_{SI}\, W_S^{\xi} r^{1-\xi},$$

where F_{SI} denotes the fixed-cost shifter for a firm from country S to sell in market I.

Using the expressions derived above, variable profits from selling to market I are given by

$$\pi_{SI}(z) = V_{SI}(z) - W_S L_{SI}(z) - rK_{SI}(z) = \frac{V_{SI}(z)}{\sigma}.$$

So the productivity cutoff $z_{\min}^{SI}$ for a firm locate at S to sell to market I is thus

$$z_{\min}^{SI} = \left(\frac{F_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}} \left(W_S^{\xi} r^{1-\xi}\right)^{\frac{\sigma}{\sigma-1}} (1 + \tau_{SI}). \tag{3}$$

Finally, I define post–fixed-cost profits from selling to market I as

$$\tilde{\pi}_{SI}(z) = \pi_{SI}(z) - F_{SI} W_S^{\xi} r^{1-\xi}.$$

2.6 Aggregated Trade Value

Aggregating across all firms located in country S that sell to market I, let Ω_{SI} denote the set of varieties—equivalently, firms—in country S that serve market I. Total trade value from S to I is then given by

$$V_{SI} = \int_{\omega \in \Omega_{SI}} V_{SI}(\omega)\, d\omega = N_S \Pr(\tilde{\pi}_{SI} > 0)\, \mathbf{E}[V_{SI}(z) \mid \tilde{\pi}_{SI} > 0]\,,$$

where N_S denotes the mass of firms in country S.

Evaluating this expression using the Pareto productivity distribution yields

$$V_{SI} = \frac{\sigma\alpha\underline{z}^{\alpha}}{\alpha+1-\sigma}\, N_S\, B_I^{\frac{\alpha}{\sigma-1}}\, F_{SI}^{1+\frac{\alpha}{1-\sigma}}\, \left(W_S^{\xi} r^{1-\xi}\right)^{1-\epsilon}\, (1+\tau_{SI})^{-\alpha}, \tag{4}$$

where $\epsilon \equiv \frac{\sigma\alpha}{\sigma-1}$.

In addition, total post–fixed-cost profits earned by firms from country S selling to market I are given by

$$\tilde{\pi}_{SI} = \int_{\omega\in\Omega_{SI}} \tilde{\pi}_{SI}(\omega)\, d\omega = \int_{\omega\in\Omega_{SI}} \left(\pi_{SI}(\omega) - F_{SI} W_S^{\xi} r^{1-\xi}\right) d\omega. \tag{5}$$

Equation (4) is standard in Melitz-type models with Pareto productivity. Relative to the canonical formulation in Melitz (2003), the only difference is that the wage term W_S is replaced by the composite unit cost $W_S^{\xi} r^{1-\xi}$, reflecting the Cobb–Douglas use of labor and capital in production.

2.7 Equilibrium Conditions

An equilibrium is a collection of wages $\{W_I\}_{I\in\{J,L\}}$, price indices $\{P_I\}_{I\in\{J,L\}}$, and measure of firms $\{N_I\}_{I\in\{J,L\}}$ such that: (i) households maximize utility subject to their budget constraints; (ii) firms choose prices, factor inputs, and market participation to maximize profits; and (iii) labor, capital, and goods markets clear, taking the rental rate of capital r as the numéraire.

Rather than working directly with the price index P_I, I use the term B_I, which is a monotone transformation of P_I, as defined in Equation 2. This transformation does not introduce an additional equilibrium object; it is merely a notational device that simplifies the expression for bilateral trade values V_{SI}.

In equilibrium, the six endogenous variables

$$\{W_J, W_L, N_J, N_L, B_J, B_L\}$$

are jointly determined by free-entry conditions in each country, labor-market–clearing conditions in each country, the global capital-market–clearing condition, and the trade-balance condition in one country (with the corresponding condition in the other country being redundant).

Free entry. For firms located in country J, the free-entry condition requires that total operating profits equal total entry costs:

$$\sum_I \tilde{\pi}_{JI} = N_J F_J^E W_J^{\xi} r^{1-\xi},$$

where F_J^E denotes the fixed-cost shifter for entry in country J, and $\tilde{\pi}_{JI}$ denotes aggregate profits earned by firms from country J in market I after deducting the fixed cost of serving that market as defined in Equation 5. Aggregate post–fixed-cost profits satisfy

$$\sum_I \tilde{\pi}_{JI} = \frac{\sigma - 1}{\sigma\alpha} \sum_I V_{JI}.$$

Substituting this expression into the free-entry condition yields

$$\frac{\sigma - 1}{\sigma\alpha} \sum_I V_{JI} = N_J F_J^E W_J^{\xi} r^{1-\xi}.$$

An analogous free-entry condition holds for firms located in country L:

$$\frac{\sigma - 1}{\sigma\alpha} \sum_I V_{LI} = N_L F_L^E W_L^{\xi} r^{1-\xi}.$$

Labor market clearing. Labor-market–clearing conditions require that the total labor endowment be allocated either to variable production or to the payment of fixed costs.

For firms located in country S, labor is used for three purposes: (i) variable production of goods shipped to destination I, (ii) the fixed cost of selling in market I, and (iii) the fixed cost of entry. Define $\tilde{L}_{SI}$ as the total labor used by firms from country S to serve destination

I, including both variable production and fixed selling costs. Specifically,

$$\tilde{L}_{SI} = N_S \int_{\omega \in \Omega_{SI}} \left[L_{SI}(\omega) + \xi F_{SI} \left(\frac{W_S}{r} \right)^{\xi - 1} \right] d\omega = \xi \frac{V_{SI}}{\sigma W_S} \left(\sigma - 1 + \frac{\alpha + 1 - \sigma}{\alpha} \right).$$

Labor-market clearing in country S therefore requires that total labor supply equal labor used for serving all destinations plus labor required for entry:

$$L_S = \sum_I \tilde{L}_{SI} + N_S F_S^E \xi \left(\frac{W_S}{r} \right)^{\xi - 1}.$$

Using the expression for $\tilde{L}_{SI}$, the labor-market–clearing conditions for each country can be written compactly as

$$L_J = \frac{\xi}{W_J} \sum_I V_{JI}, \qquad L_L = \frac{\xi}{W_L} \sum_I V_{LI}.$$

Capital market clearing. With frictionless capital mobility, there is a single integrated capital market. The capital-market–clearing condition requires that the total capital endowment be allocated either to variable production or to the payment of fixed costs.

As with labor, capital is used for three purposes: (i) variable production of goods shipped to destination I, (ii) the fixed cost of selling in market I, and (iii) the fixed cost of entry. Define $\tilde{K}_{JI}$ as the total capital used by firms from country J to serve destination I, including both variable production and fixed selling costs. Analogous definitions apply for firms located in country L.

Under this notation, the aggregate capital-market–clearing condition is

$$K_0 = \sum_I \tilde{K}_{JI} + \sum_I \tilde{K}_{LI} + N_J F_E^J (1 - \xi) \left(\frac{W_J}{r} \right)^{\xi} + N_L F_E^L (1 - \xi) \left(\frac{W_L}{r} \right)^{\xi}.$$

Note that

$$\sum_I \tilde{K}_{JI} + N_J F_E^J (1 - \xi) \left(\frac{W_J}{r} \right)^{\xi}$$

represents the total capital used by firms located in country J. This generally differs from $K_J = \gamma K_0$, which denotes capital ownership. With frictionless capital mobility, ownership and usage need not coincide.

The total capital used by firms from country J to serve destination I is given by

$$\tilde{K}_{JI} = N_J \int_{\omega \in \Omega_{JI}} \left[K_{JI}(\omega) + (1-\xi) F_{JI} \left(\frac{W_J}{r} \right)^{\xi} \right] d\omega = \frac{1-\xi}{\sigma r} V_{JI} \left(\sigma - 1 + \frac{\alpha + 1 - \sigma}{\alpha} \right).$$

Substituting this expression into the capital-market–clearing condition yields

$$K_0 = \frac{1-\xi}{r} \sum_I \left(V_{JI} + V_{LI} \right).$$

Goods market clearing and trade balance. Combining the labor- and capital-market–clearing conditions implies

$$W_J L_J + W_L L_L + r K_0 = \sum_I V_{JI} + \sum_I V_{LI},$$

which states that total expenditure equals total income in the world economy.

Trade-balance conditions require that total expenditure equals total income within each country. For country J,

$$W_J L_J + \gamma r K_0 = V_{JJ} + V_{LJ},$$

and for country L,

$$W_L L_L + (1-\gamma) r K_0 = V_{LL} + V_{JL}.$$

Number of firms. Combining the country-specific free-entry condition with the labor-market–clearing condition yields an expression for the equilibrium mass of firms that is monotonically related to country-specific wages:

$$N_S = \frac{L_S}{\xi F_S^E} \frac{\sigma - 1}{\sigma \alpha} \left(\frac{W_S}{r} \right)^{1-\xi}. \tag{6}$$

Since N_S is a function of W_S, the number of endogenous variables can be reduced from

$\{W_J, W_L, B_J, B_L, N_J, N_L\}$ to $\{W_J, W_L, B_J, B_L\}$. Accordingly, the two free-entry conditions can be substituted out when solving for equilibrium.

Summary. The equilibrium is characterized by the following four equations:

$$L_J = \frac{\xi}{W_J} \sum_I V_{JI}, \tag{7}$$

$$L_L = \frac{\xi}{W_L} \sum_I V_{LI}, \tag{8}$$

$$K_0 = \frac{1-\xi}{r} \sum_I \left(V_{JI} + V_{LI}\right), \tag{9}$$

$$\mu_J W_J L_J + \gamma r K_0 = V_{JJ} + V_{LJ}. \tag{10}$$

Here, bilateral sales V_{SI} are obtained by substituting Equation 6 into Equation 4, yielding

$$V_{SI} = \frac{(\sigma-1)\underline{z}^{\alpha}}{\alpha+1-\sigma} \frac{L_S}{\xi F_S^E} \left(\frac{W_S}{r}\right)^{1-\xi} B_I^{\frac{\alpha}{\sigma-1}} F_{SI}^{1+\frac{\alpha}{1-\sigma}} \left(W_S^{\xi} r^{1-\xi}\right)^{1-\epsilon} (1+\tau_{SI})^{-\alpha}. \tag{11}$$

Appendix A solves for the equilibrium under symmetry between the two countries. In the symmetric equilibrium,

$$V_{JL} = V_{LJ}, \qquad V_{JJ} = V_{LL}, \qquad W_J = W_L, \qquad B_J = B_L, \qquad N_J = N_L.$$

This symmetric equilibrium serves as the baseline for the counterfactual analysis.

2.8 Wage–Entry Linkage

Equation 6, shows that the equilibrium number of firms responds to endogenous wage changes, rather than being fully pinned down by exogenous parameters. This wage–entry linkage is novel. In the standard Melitz (2003) framework, the equilibrium mass of firms depends only on labor endowments (e.g., L_I) and model parameters[1]

The wage–entry linkage emerges specifically from the interaction between capital usage and capital mobility. When firm entry is endogenously determined by the free-entry condition, two general implications arise. First, the number of firms is given by

$$N = \frac{\text{total profit}}{\text{fixed cost}},$$

which in this paper takes the form

$$N_S = \frac{\sum_I \tilde{\pi}_{SI}}{F_S^E W_S^{\xi} r^{1-\xi}}.$$

Second, while the cost function remains homogeneous of degree one in factor prices, it is no longer homogeneous of degree one in quantities due to the presence of fixed costs.

Since factor prices enter the cost function homothetically, and total profit is proportional to cost, the number of firms is determined by relative factor prices, which in this paper are summarized by the wage–rental ratio $\frac{W_I}{r}$.

When all factors are cleared domestically (as in Melitz (2003)), relative factor prices are pinned down by domestic endowments and model parameters. As shown in Appendix B, with two immobile factors,

$$\frac{r_S}{W_S} = \frac{1-\xi}{\xi} \frac{L_S}{K_S}, \tag{12}$$

where ξ, K_S, and L_S are all exogenous.

However, when one factor is mobile, the two factor prices are no longer perfectly linked, such that

$$\frac{r}{W_S} = \frac{1-\xi}{\xi} \frac{L_S}{K_0} \frac{\sum_S \sum_I V_{SI}}{\sum_I V_{SI}}. \tag{13}$$

[1]This linkage does not arise mechanically from introducing capital into production. As shown in Appendix B, when capital is used in production but remains internationally immobile, the equilibrium mass of firms continues to be pinned down by exogenous parameters and is invariant to endogenous wage movements. Appendix C further shows that allowing fixed costs to differ in factor intensities does not alter this result. Finally, Appendix D demonstrates that specifying fixed costs as constants likewise leaves the mechanism unchanged.

Here, ξ, K_0, and L_S are exogenous parameters, while $\frac{\sum_I V_{SI}}{\sum_S \sum_I V_{SI}}$ is endogenously determined and represents country S's share of world revenue. From the labor market clearing conditions (Equations 7 and 8), this share can be written as

$$\frac{\sum_I V_{SI}}{\sum_S \sum_I V_{SI}} = \frac{W_S L_S}{\sum_S W_S L_S}. \tag{14}$$

In the symmetric two-country case, this expression reduces to

$$\frac{r}{W_J} = \frac{1-\xi}{\xi} \frac{L_S}{K_0} \left(1 + \frac{W_L}{W_J}\right). \tag{15}$$

Equation (15) shows that the rental–wage ratio is decreasing in country J's wage relative to country L's wage. When W_J/W_L increases, country J captures a larger share of world revenue, which lowers its relative rental price. Intuitively, higher wages are associated with a larger domestic market and higher GDP, which attracts a greater share of mobile capital for production in country J, regardless of capital ownership. This increase in effective capital availability reduces the relative rental rate, making firm entry cheaper and leading to a larger equilibrium mass of firms in country J.

2.9 Comparative Statics

Given the symmetric baseline solution, I study comparative statics with respect to changes in iceberg trade costs. My primary goal is to analyze how an exogenous change in trade costs influences equilibrium outcomes.

2.9.1 Log-differentiating the trade values

Log-differentiating Equation 6 yields

$$\hat{N}_S = (1-\xi)\hat{W}_S. \tag{16}$$

Thus, changes in the mass of firms are positively correlated with changes in wages.

At the same time, log-differentiating the trade value equation (Equation 4) yields

$$\hat{V}_{SI} = \hat{N}_S + (1-\epsilon)\xi \hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha \hat{\tau}_{SI}. \tag{17}$$

Substituting Equation 16 into Equation 17 gives

$$\hat{V}_{SI} = (1-\xi)\hat{W}_S + (1-\epsilon)\xi \hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha \hat{\tau}_{SI}.$$

Collecting terms in $\hat{W}_S$ yields

$$\hat{V}_{SI} = (1-\xi\epsilon)\hat{W}_S + \frac{\alpha}{\sigma - 1}\hat{B}_I - \alpha \hat{\tau}_{SI}. \tag{18}$$

The elasticity of sales with respect to wages in response to a tariff change is therefore $1 - \xi\epsilon$, which can be either positive or negative, and is more likely to be positive when ξ is small. In contrast, in canonical trade models this elasticity is always negative.[2]

2.9.2 Log-differentiating the equilibrium conditions

Given the generalized log-differentiated expression for sales revenue, I now log-differentiate the equilibrium conditions.

Labor market clearing conditions. Log-differentiating the labor market clearing conditions yields:

$$\sum_I V_{SI}\,\hat{W}_S = \sum_I V_{SI}\,\hat{V}_{SI}, \tag{19}$$

Substituting Equation 18 into the log-linearized labor market clearing conditions above gives:

$$\sum_I V_{SI}\hat{W}_S = (1-\xi\epsilon)\sum_I V_{SI}\hat{W}_S + \frac{\alpha}{\sigma - 1}\sum_I V_{SI}\hat{B}_I - \alpha \sum_I V_{SI}\hat{\tau}_{SI}, \tag{20}$$

[2] See Appendix E for further discussion of the differences.

Capital market clearing conditions. Log-linearizing the capital market clearing condition in Equation 9 yields:

$$\sum_I V_{JI}\hat{V}_{JI} + \sum_I V_{LI}\hat{V}_{LI} = 0. \tag{21}$$

Using Equations 19, Equation 21 can be rewritten as:

$$\sum_I V_{JI}\hat{W}_J + \sum_I V_{LI}\hat{W}_L = 0. \tag{22}$$

The capital market clearing condition therefore implies that equilibrium wage changes in the two countries must move in opposite directions; in the symmetric-country case, they move in opposite directions with equal magnitude from the baseline equilibrium.

Trade balance conditions. Log-differentiating the trade balance condition for country J yields:

$$W_J L_J \hat{W}_J = V_{JJ}\hat{V}_{JJ} + V_{LJ}\hat{V}_{LJ}.$$

Substituting Equation 18 into this expression gives:

$$\big(\xi \sum_I V_{JI} + V_{JJ}(\xi\epsilon - 1)\big)\hat{W}_J + (\xi\epsilon - 1)V_{LJ}\hat{W}_L + \frac{\alpha(V_{JJ} + V_{LJ})}{1-\sigma}\hat{B}_J = -\alpha V_{LJ}\hat{\tau}_{LJ}. \tag{23}$$

Matrix representation. The log-linearized equilibrium conditions can be summarized in the following general matrix form. In this representation, $\hat{N}_J$ and $\hat{N}_L$ have been substituted out using Equations 16, so that the system is expressed solely in terms of $\hat{W}_J$, $\hat{W}_L$, and $\hat{B}_J$, $\hat{B}_L$.[3]

$$\begin{pmatrix} \xi\epsilon \sum_I V_{JI} & 0 & \frac{\alpha}{1-\sigma}V_{JJ} & \frac{\alpha}{1-\sigma}V_{JL} \\ 0 & \xi\epsilon \sum_I V_{LI} & \frac{\alpha}{1-\sigma}V_{LJ} & \frac{\alpha}{1-\sigma}V_{LL} \\ \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\ \xi \sum_I V_{JI} + (\xi\epsilon - 1)V_{JJ} & (\xi\epsilon - 1)V_{LJ} & \frac{\alpha}{1-\sigma}(V_{JJ} + V_{LJ}) & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

[3] A version of the matrix system that explicitly includes $\hat{N}_J$ and $\hat{N}_L$ is provided in Appendix F.

$$= \begin{pmatrix} -\alpha V_{JL}\hat{\tau}_{JL} \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \end{pmatrix}$$

I further simplify the expression by defining

$$\lambda \equiv \frac{V_{JL}}{V_{JL} + V_{JJ}},$$

which represents the share of country J's GDP derived from exports. Using the symmetry of the equilibrium, this expression can be rewritten as

$$\begin{pmatrix} \xi\epsilon & 0 & \frac{\alpha(1-\lambda)}{1-\sigma} & \frac{\alpha}{1-\sigma}\lambda \\ 0 & \xi\epsilon & \frac{\alpha\lambda}{1-\sigma} & \frac{\alpha}{1-\sigma}(1-\lambda) \\ 1 & 1 & 0 & 0 \\ \xi + (\xi\epsilon - 1)(1-\lambda) & (\xi\epsilon - 1)\lambda & \frac{\alpha}{1-\sigma} & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha\lambda\hat{\tau}_{JL} \\ -\alpha\lambda\hat{\tau}_{LJ} \\ 0 \\ -\alpha\lambda\hat{\tau}_{LJ} \end{pmatrix}$$

2.9.3 Bilateral trade liberalization

Let us first consider a symmetric bilateral trade liberalization in which both countries reduce bilateral trade costs by the same amount,

$$\hat{\tau}_{JL} = \hat{\tau}_{LJ} \equiv \hat{\tau} < 0.$$

Solving the system of log-linearized equilibrium conditions yields

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \lambda(\sigma-1)\hat{\tau} \\ \lambda(\sigma-1)\hat{\tau} \end{pmatrix}.$$

A bilateral trade liberalization therefore does not affect wages in either country relative to the rental rate of capital. As a result, nominal household income remains unchanged in both countries. However, the trade liberalization lowers the price index through changes in B_J and B_L, and this price index adjustment is identical across countries under symmetry.

Household welfare changes in country $I \in \{J, L\}$ are given by changes in real income,

$$(\frac{\hat{W}_I W_I L_I + \hat{r} r K_I}{W_I L_I + r K_I}) - \hat{P}_I.$$

Since the rental rate is the numeraire, $\hat{r} = 0$, so welfare changes reduce to

$$\frac{W_I L_I}{W_I L_I + r K_I} \hat{W}_I - \hat{P}_I = s_I \hat{W}_I - \hat{P}_I.$$

Here, s denotes the labor income share,

$$s_I \equiv \frac{W_I L_I}{W_I L_I + r K_I},$$

which equals ξ in the symmetric benchmark.

Proposition 1 *Under bilateral trade liberalization from symmetry, real income in each country $I \in \{J, L\}$ increases by the same amount:*

$$s_I \hat{W}_I - \hat{P}_I = -\hat{P}_I = -\frac{1}{\sigma - 1}\big(\hat{B}_I - s_I \hat{W}_I\big) = -\lambda\hat{\tau} > 0.$$

2.9.4 Unilateral trade liberalization

Consider a unilateral trade liberalization in which only country J reduces the trade barrier applied to imports from country L. That is, the trade cost for goods shipped from L to J falls, while the trade cost from J to L remains unchanged:

$$\hat{\tau}_{JL} = 0, \qquad \hat{\tau}_{LJ} = \hat{\tau} < 0.$$

Solving the system of log-linearized equilibrium conditions yields

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} \frac{\alpha\lambda(1-\lambda)}{A}\,\hat{\tau} \\ -\frac{\alpha\lambda(1-\lambda)}{A}\,\hat{\tau} \\ (\sigma-1)\frac{\lambda C}{A}\,\hat{\tau} \\ -(\sigma-1)\frac{(1-\lambda)D}{A}\,\hat{\tau} \end{pmatrix}.$$

where

$$A = (1-\xi) + 2\lambda\xi + 4\lambda(1-\lambda)(\epsilon\xi - 1) > 0,$$

$$C = \lambda\big(\xi - (1-2\lambda)(1-\epsilon\xi)\big) + \epsilon\xi,$$

$$D = \lambda\big(\xi - (1-2\lambda)(1-\epsilon\xi)\big).$$

Proposition 2 *Under unilateral trade liberalization by country J from symmetry, if the export share of GDP satisfies $\lambda < \frac{1}{2}$, nominal wages in country J decrease, while nominal wages in country L increase by the same magnitude.*

Proof. Since $\hat{\tau} < 0$, the sign of the wage response in country J is determined by the sign of the coefficient $\alpha\lambda(1-\lambda)/A$. For $\lambda \in (0,1)$, the numerator is strictly positive. As shown in Appendix G, the denominator A is strictly positive when $\lambda < \frac{1}{2}$, a condition that is empirically plausible. Therefore, $\hat{W}_J < 0$. By the capital market clearing condition, equilibrium wage changes must have equal magnitude and opposite signs, implying $\hat{W}_L = -\hat{W}_J > 0$. ■

Implications for GDP. Using Proposition 2, we now characterize the change in GDP for country J under unilateral trade liberalization. Let total GDP in country J be the value of domestic sales plus exports. Log-differentiating yields

$$\hat{V}_J = (1-\lambda)\hat{V}_{JJ} + \lambda \hat{V}_{JL}. \tag{24}$$

Substituting Equation 18 into this expression gives

$$\hat{V}_J = (1-\lambda)(1-\xi\epsilon)\hat{W}_J + \frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J + \lambda(1-\xi\epsilon)\hat{W}_L + \frac{\alpha\lambda}{\sigma-1}\hat{B}_L. \tag{25}$$

Using the fact that $\hat{W}_J = -\hat{W}_L$, this expression can be rewritten as

$$\hat{V}_J = (1-\xi\epsilon)\hat{W}_J + \frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J + \frac{\alpha\lambda}{\sigma-1}\hat{B}_L. \tag{26}$$

Substituting the closed-form solutions for $\hat{B}_J$ and $\hat{B}_L$ shows that

$$\frac{\alpha(1-\lambda)}{\sigma-1}\hat{B}_J = C\,\hat{W}_J, \qquad \frac{\alpha\lambda}{\sigma-1}\hat{B}_L = -D\,\hat{W}_J,$$

Equation 26 therefore simplifies to

$$\hat{V}_J = \big[(1-\xi\epsilon) + (C-D)\big]\hat{W}_J, \tag{27}$$

which reduces to

$$\hat{V}_J = \hat{W}_J. \tag{28}$$

Given that $\hat{W}_J$ increases with an increase in $\hat{\tau}_{LJ}$, a unilateral increase in import tariffs raises domestic GDP in country J.

Moreover, an increase in GDP in country J implies a higher demand for capital. Since capital used in country J satisfies

$$\tilde{K}_J = (1-\xi)V_J,$$

it follows that

$$\hat{\tilde{K}}_J = \hat{V}_J.$$

Therefore, a unilateral tariff increase induces a reallocation of capital toward country J, drawing capital away from the foreign country. In Appendix B, Paragraph B.6, I show that, in the absence of capital mobility, domestic GDP is invariant to trade-cost changes of any kind. The endogenous response of domestic GDP is one of the mechanism that is distinct in this paper compared to the literature..

Implications for the Price Index Substituting the solution into the log-change expression for the price index,

$$\hat{P}_I = \frac{1}{\sigma - 1}\left(\hat{B}_I - \xi \hat{W}_I\right), \tag{29}$$

yields

$$\hat{P}_J = \frac{(\lambda+\sigma-1)\hat{V}_J - (\lambda+\sigma-1)\hat{N}_J - \lambda \hat{B}_L}{(\sigma-1)(1-\lambda)}. \tag{30}$$

For comparison, in the benchmark case with immobile capital, the domestic price index reduces to

$$\hat{P}_J = -\frac{\lambda}{1-\lambda}\hat{B}_L, \tag{31}$$

as shown in Equation 83. In this case, the domestic price index responds solely to changes in the foreign market size B_L.

With capital mobility, the domestic price index is no longer a function only of foreign market size. The presence of the factor $\frac{1}{\sigma-1}$ further implies that the influence of foreign market size is attenuated. In addition to the term $\hat{B}_L$, the price index depends on domestic aggregate production and the domestic mass of firms. The coefficient on $\hat{V}_J$ is positive, consistent with higher domestic production being associated with higher equilibrium wages and production costs, and thus a higher price index. In contrast, the coefficient on $\hat{N}_J$ is negative, indicating that an increase in the number of domestic firms lowers the price index, as consumers substitute toward a larger set of domestically produced varieties that are not subject to iceberg trade costs. As a result, increased firm entry partially offsets the price-increasing effect of higher domestic output. Overall, the elasticity of the domestic price index with respect to a unilateral tariff change is smaller than in the case of immobile capital. Under normal circumstances, a unilateral tariff increase reduces foreign market size and therefore raises the domestic price index; however, in the present model, this price effect is quantitatively weaker.

An abnormal case may also arise in this model because the term $1 - \epsilon\xi$ is not necessarily negative. If $1 - \epsilon\xi > 0$ and its magnitude is sufficiently large—an outcome that is extremely rare and occurs only for very small values of ξ (e.g., $\xi < 0.05$)—foreign market size may increase following a unilateral tariff increase. The underlying mechanism is that a sufficiently large capital inflow into the domestic country J leads to a sharp decline in the foreign mass of firms, forcing foreign consumers to substitute toward goods produced in country J. In this case, the domestic price index may decrease in response to a unilateral tariff increase.

I now further characterize the conditions under which the abnormal case arises. Substituting the solution into the expression for the change in the domestic price index yields

$$\hat{P}_I = \frac{\lambda}{A}\left(C - \xi\frac{\alpha(1-\lambda)}{\sigma - 1}\right)\hat{\tau} = \frac{\lambda}{A}F\hat{\tau}. \tag{32}$$

When $F > 0$, a unilateral tariff reduction leads to a decrease in the domestic price index, generating welfare gains for investors. In contrast, when $F < 0$, the domestic price index increases following a unilateral trade liberalization, as in the Metzler paradox, and investors experience welfare losses.

The condition for $F < 0$ is equivalent to

$$\lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big)+\xi\frac{\alpha(1-\lambda)}{\sigma-1} > \epsilon\xi. \tag{33}$$

where

$$F = C - \xi\frac{\alpha(1-\lambda)}{\sigma-1},$$

Since $\xi\frac{\alpha(1-\lambda)}{\sigma-1} > 0$, a sufficient condition for $F < 0$ is $C < 0$, which gives

$$C < 0 \iff \lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big) > \epsilon\xi. \tag{34}$$

By continuity, the left-hand side of inequality (34) is decreasing in ξ. By the Intermediate Value Theorem, there exists a unique threshold $\xi^* \in (0,1)$ such that the inequality holds with equality. It follows that $C < 0$ if and only if $\xi < \xi^*$. Since $C < 0$ is a sufficient condition for $F < 0$, there therefore exist values of ξ for which $F < 0$ holds.

In general, smaller values of ϵ and ξ make it more likely that $F < 0$, implying a greater likelihood that the domestic price index increases in response to a unilateral tariff reduction.

Welfare Implications The change in household welfare is given by

$$\xi\hat{W}_I - \hat{P}_I = \xi\left(1+\frac{\xi}{\sigma-1}\right)\hat{W}_I - \frac{\hat{B}_I}{\sigma-1}.$$

Households experience welfare losses from unilateral trade liberalization if the following condition is satisfied:

$$\xi\left(1+\frac{\xi}{\sigma-1}\right)\alpha(1-\lambda)+\lambda\big((1-2\lambda)(1-\epsilon\xi)-\xi\big) > \epsilon\xi. \tag{35}$$

Since $\alpha(1-\lambda) > 0$, Condition 35 is less restrictive than Condition 33. Consequently, the set of parameter values satisfying Condition 35 strictly contains those satisfying Condition 33.

It follows that there exist two thresholds $\bar{\xi}_1 < \bar{\xi}_2$ such that, for all $\xi \in (\bar{\xi}_1, \bar{\xi}_2)$, the domestic price index decreases under trade liberalization while households still experience welfare losses.

Implications for the export share of GDP (λ) Finally, I want to analytically discuss the change of export share of GDP, $\hat{\lambda}$ under the unilateral tariff change. Log-differentiating $\lambda = \frac{V_{JL}}{V_{JJ}+V_{JL}}$ under unilateral trade liberalization (i.e., $\hat{\tau}_{JL} = 0$) gives:

$$\hat{\lambda} = (1-\lambda)(\hat{V}_{JL} - \hat{V}_{JJ}) = (1-\lambda)\frac{\alpha}{\sigma-1}(\hat{B}_L - \hat{B}_J) \tag{36}$$

Substituting $\hat{V}_{SI}$ from Equation 11, I obtain:

$$\hat{\lambda} = -\frac{\alpha\lambda(1-\lambda)\hat{\tau}}{A}(\xi - (1-2\lambda)(1-\epsilon\xi) + \epsilon\xi). \tag{37}$$

The condition for $\hat{\lambda}$ to increase with a unilateral tariff increase (or decrease with a unilateral trade liberalization increase) is thus:

$$(1-2\lambda)(1-\epsilon\xi) - \xi > \epsilon\xi. \tag{38}$$

Compare the condition 38 with condition 34, one can clearly see that condition 34 implies condition 38, however, the ordering between condition 35 and condition 38 is not very obvious. In the following proposition, I prove that there is actually a clear order between those conditions.

Proposition 3 *Condition 33 implies Condition 38, which in turn implies Condition 35.*

Proof. Proof can be found in Appendix H. ■

Proposition 3 shows that households may experience welfare losses from unilateral trade liberalization even when such a policy increases the export share of GDP and lowers the domestic price index. More concretely, even if the data indicate that domestic firms are exporting more to foreign markets and consumers face lower prices, households can still suffer welfare losses.

3 Multi-Region Calibration

With the intution from a two-country framework in-hand, this section extends the two-country environment to an n-region setting.

3.1 Equilibrium Conditions

Consider n regions indexed by $S \in \{1, \dots, n\}$. Each region S is endowed with labor L_S and a share γ_S of the aggregate capital endowment K_0, where $\sum_{S=1}^{n} \gamma_S = 1$. Let V_{SI} denote total sales in destination I by firms based in region S. The equilibrium is characterized by the following conditions.

n **Free entry conditions.** For each origin region S, the free-entry condition is

$$\frac{\sigma - 1}{\sigma \alpha_S} \sum_{I=1}^{n} V_{SI} = N_S F_S^E W_S^{\xi} r^{1-\xi}. \tag{39}$$

n **Labor-market clearing conditions.** For each region S,

$$L_S = \frac{\xi}{W_S} \sum_{I=1}^{n} V_{SI}. \tag{40}$$

$n - 1$ **Trade balance conditions.** For each region S, imposed for $n - 1$ regions,

$$W_S L_S + \gamma_S r K_0 = \sum_{I=1}^{n} V_{IS}. \tag{41}$$

one Global capital-market clearing condition. Aggregate capital demand equals the world capital endowment:

$$K_0 = \frac{1-\xi}{r} \sum_{S=1}^{n} \sum_{I=1}^{n} V_{SI}. \tag{42}$$

3.2 Exact Hat Algebra

Suppose τ_{SI} is the policy variable, applying exact hat methods, where $\hat{X} = \frac{X'}{X}$, The exact hat of sale from source region S to desination region I is:

$$\hat{V}_{SI} = \hat{N}_S \hat{B}_I^{\frac{\alpha}{\sigma-1}} \hat{W}_S^{\xi(1-\epsilon)} \widehat{1+\tau_{SI}}^{-\alpha} \tag{43}$$

where

$$\hat{N}_S = \hat{W}_S^{1-\xi} \tag{44}$$

and the market clearing conditions are thus, for each source region S,

$$\sum_I V_{SI} \hat{W}_S = \sum_I V_{SI} \hat{V}_{SI} \tag{45}$$

For each destination region I

$$W_I L_I \hat{W}_I + \gamma_I r K_0 = \sum_S V_{SI} \hat{V}_{SI} \tag{46}$$

and the one capital market clearing condition as:

$$rK_0 = (1-\xi) \sum_S \sum_I V_{SI} \hat{W}_S \tag{47}$$

3.3 2018 Tariff Policy

I analyze the welfare consequences of President Trump's 2018 tariff policy toward China using the exact-hat algebra implied by the multi-region model. The policy is modeled as an increase in tariffs imposed by the 51 U.S. regions (the 50 states and the Wachingtong D.C) on imports from China, from an average tariff rate of 1.66% to 25%. Chinese retaliation is modeled as a symmetric increase in China's tariffs on imports from the United States to 25%.

3.3.1 Calibration

The parameters required to solve the exact-hat equilibrium are

$$\Theta = \Big\{\sigma,\ \alpha,\ \xi,\ \gamma_S \in \mathbb{R}^{52},\ \tau_{SI} \in \mathbb{R}^{52\times 52}\Big\}.$$

Table 1 reports the parameter values used in the calibration. Following Broda and Weinstein (2006), I set the elasticity of substitution to $\sigma = 3$. In Melitz-type models, the ratio $\alpha/(\sigma-1)$ governs the shape of the firm-size distribution. Empirical estimates reported in Akgul and Ahmad (2020) suggest that this ratio typically lies between 1.5 and 3.5 across industries. Given $\sigma = 3$, I choose $\alpha = 4$, which implies $\frac{\alpha}{\sigma-1} = 2$. Finally, I set $\xi = 0.6$, consistent with values commonly used in the quantitative trade literature.

Parameter	**Value**
σ	3
α	4
ξ	0.6

Table 1: Parameter values used in the calibration.

I construct a $(52 \times 52 - 1)$ trade-value matrix V using trade data for 51 U.S. regions and China for the year 2017. State-to-state trade flows within the United States are obtained from the Freight Analysis Framework (FAF).[4] State-level exports to and imports from China

[4] U.S. Department of Transportation, Bureau of Transportation Statistics (BTS), and Federal Highway Administration (FHWA), *Freight Analysis Framework (FAF), Version 5*, 2017–2050. DOI: 10.21949/1529116.

are constructed using data from USA Trade Online.[5] China's domestic sales are not directly observed. To complete the trade matrix, I impute China's domestic sales using the relative size of the Chinese and U.S. economies[6]:

$$V_{CHN,CHN} + \sum_{I \neq CHN} V_{CHN,I} = \left(\sum_{S \in \text{US}} \sum_{I} V_{S,I} \right) \frac{GDP_{CHN}}{GDP_{US}} \tag{48}$$

where GDP_{CHN} and GDP_{US} denote China's GDP and aggregate U.S. GDP, respectively, obtained from the World Bank.

To assess the validity of the constructed trade matrix, I compare matrix-implied GDP (row sums) with BEA-reported regional GDP in 2017 and matrix-implied personal consumption expenditures (PCE) (column sums) with BEA-reported regional PCE in 2017. Figure 3 presents the comparison excluding China, while Figure 4 includes China. The strong correlation between the values implied by the constructed trade matrix and the corresponding reported statistics suggests that the trade matrix provides a reasonable representation of regional economic activity and consumption patterns, supporting its use in the exact hat algebra counterfactual analysis.

For bilateral iceberg trade costs (tariffs) τ_{SI}, I assume that there are no tariffs on trade within the United States. For U.S. exports to China, I use the average Chinese tariff on U.S. goods in 2017 (0.0383) and assign this value uniformly across U.S. states. Similarly, for Chinese exports to the United States, I use the average U.S. tariff applied to Chinese goods in 2017 (0.0166).

Finally, for the region-specific capital shares γ_S, the equilibrium conditions imply

$$\gamma_S = \frac{\sum_S V_{SI} - \xi \sum_I V_{SI}}{(1-\xi) \sum_S \sum_I V_{SI}}. \tag{49}$$

[5]U.S. Census Bureau, *USA Trade Online*, https://usatrade.census.gov/.

[6]Domestic trade flows are commonly constructed as GDP minus exports in the structural gravity literature (e.g., Head and Mayer (2014)). Because my dataset contains only bilateral trade between the United States and China and excludes their trade with other partners, this accounting identity cannot be applied directly. I therefore approximate China's total sales using relative GDP sizes and recover domestic sales as the residual.

Thus, γ_S can be computed directly from the trade matrix using the row sums, column sums, and the sum of all entries. Alternatively, one can compute γ_S using regional GDP, regional personal consumption expenditure shares, and aggregate GDP across all regions, since row sums proxy for regional GDP, column sums proxy for personal consumption expenditures, and the sum of all entries proxies for total GDP. The resulting values of γ_S are very similar across the two approaches.

3.4 Counterfactual Number of Firms Effects

One important channel in this model is that trade policy can affect firm entry. Considering that firm entry or the creation of new business establishments may take time, I regress the percentage change in the number of establishments across 51 U.S. regions in subsequent years, measured using County Business Patterns (CBP) data, on the model-predicted percentage change in the number of firms. Specifically, $\hat{N}_{yr} = \frac{N_{yr} - N_{2017}}{N_{2017}}$. The results are reported in Table 2 and Table 3. Although the model does not explain all of the variation in establishment growth across regions, the regression results indicate that the cross-regional percentage change in the number of firms predicted by the model is positively correlated with the percentage change in the number of establishments reported in the CBP data.

Table 2: Without Retaliation

	(1)	(2)	(3)	(4)	(5)
	$\hat{N}_{2018}$	$\hat{N}_{2019}$	$\hat{N}_{2020}$	$\hat{N}_{2021}$	$\hat{N}_{2022}$
$\hat{N}_{\text{model}}$	1.472	2.809	5.384	8.813*	12.467**
	(1.292)	(1.960)	(3.585)	(4.645)	(5.950)
Constant	-0.014	-0.025	-0.053	-0.077	-0.103
	(0.016)	(0.024)	(0.043)	(0.055)	(0.071)
Observations	51	51	51	51	51
R-squared	0.019	0.025	0.028	0.036	0.046
Adj. R-squared	-0.001	0.005	0.008	0.016	0.027

Notes: Robust standard errors in parentheses, computed using the HC3 estimator. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 3: With Retaliation

	(1)	(2)	(3)	(4)	(5)
	$\hat{N}_{2018}$	$\hat{N}_{2019}$	$\hat{N}_{2020}$	$\hat{N}_{2021}$	$\hat{N}_{2022}$
$\hat{N}_{\text{model}}$	1.070	2.002	4.242	6.923*	9.427**
	(1.144)	(1.762)	(3.123)	(3.800)	(4.692)
Constant	-0.007	-0.012	-0.031	-0.041	-0.048
	(0.011)	(0.018)	(0.031)	(0.037)	(0.046)
Observations	51	51	51	51	51
R-squared	0.012	0.015	0.021	0.027	0.032
Adj. R-squared	-0.008	-0.005	0.001	0.007	0.012

Notes: Robust standard errors in parentheses, computed using the HC3 estimator. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

3.5 Counterfactual Welfare Effects

Figure 1 shows the cross regional welfare consequences of a unilateral 23% ($1.25/1.016 \approx 1.23$) increase and a bilateral 23% tariff increase for pure wage earners, hereafter referred to as workers. In both scenarios, workers in all U.S. regions experience welfare gains. When it comes to a region representative household which is assumed to earn both labor and capital income, the cross regional welfare change effect are shown in Figure 2.

With the rental rate normalized as the numeraire, the larger the share of rental income in household income, the more household welfare is driven by changes in the price index. Households that rely entirely on rental income do not benefit from wage changes; their welfare is determined solely by movements in the price index.

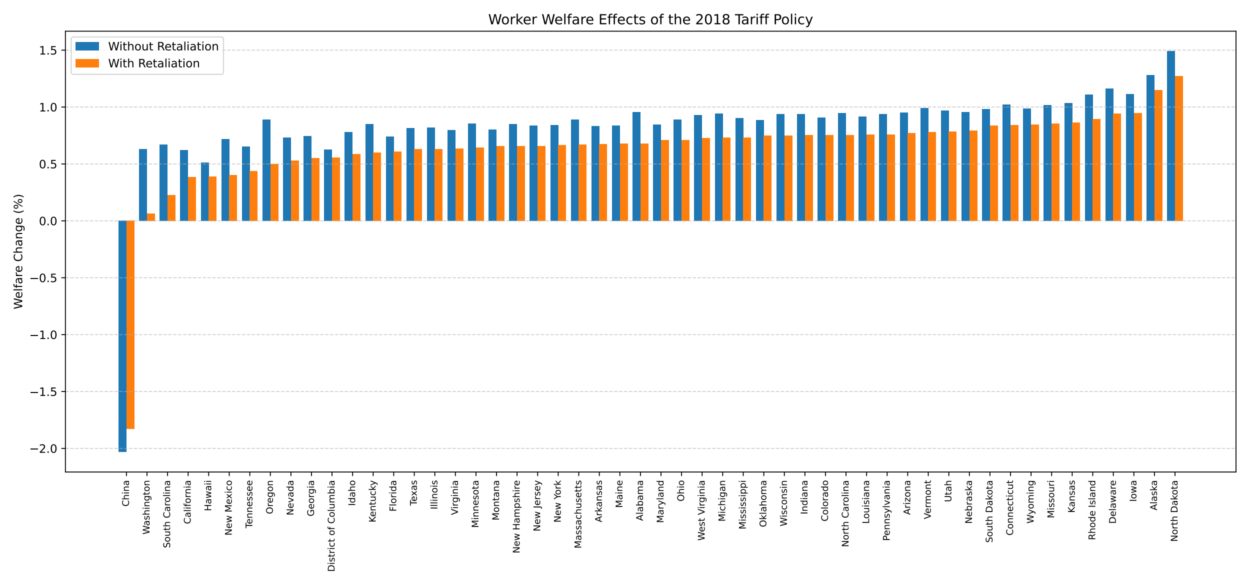

Figure 1: Workers welfare changes with vs without retaliation.

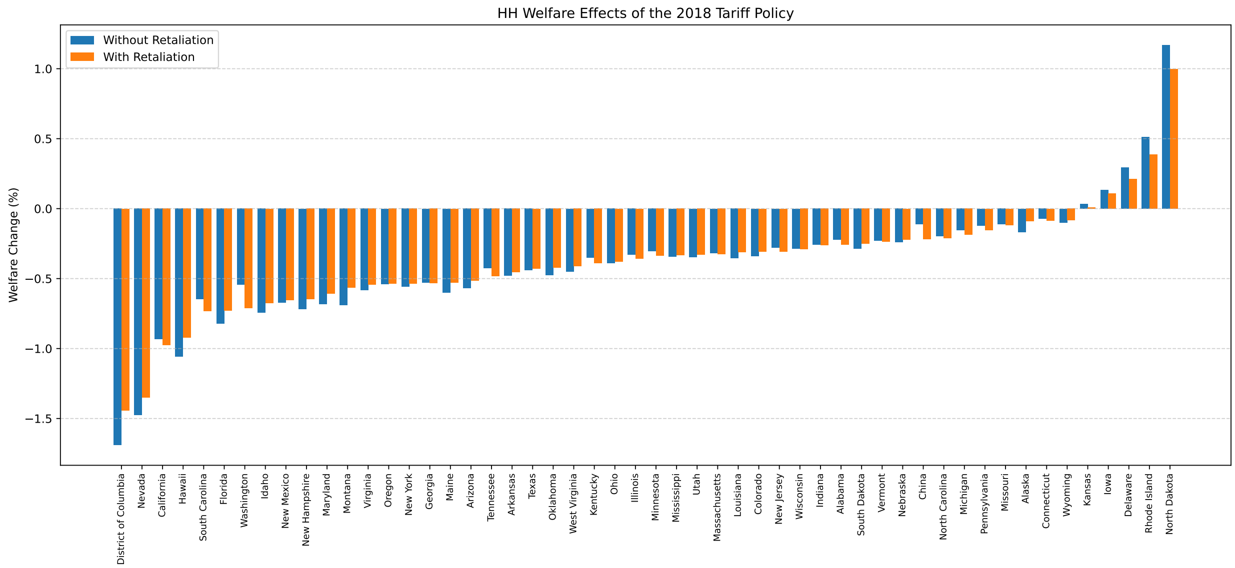

Figure 2: HH welfare changes with vs without retaliation.

3.6 Connection to Voting Outcomes

Finally, I examine the relationship between the model-predicted welfare change and the Republican vote share in the 2016 election.

Table 4: Republican Vote Share vs Model Predicted State Welfar Consequences

	(1)	(2)	(3)	(4)	(5)	(6)
%Δ HH welfare	11.507*					
	(6.032)					
%Δ HH welfare with retaliation		13.238**				
		(6.420)				
%Δ Worker welfare			26.309**			
			(10.384)			
%Δ Worker welfare with retaliation				19.464***		
				(7.010)		
%Δ Melitz worker welfare					-5.333	
					(6.357)	
Δ Melitz worker welfare with retaliation						-6.819
						(10.203)
Constant	0.529***	0.535***	0.250**	0.350***	0.482***	0.479***
	(0.024)	(0.025)	(0.099)	(0.056)	(0.018)	(0.022)
Observations	51	51	51	51	51	51
R-squared	0.178	0.185	0.140	0.108	0.055	0.039
Adj. R-squared	0.161	0.168	0.122	0.090	0.036	0.020

Notes: Robust standard errors in parentheses, computed using the HC3 estimator. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

4 Conclusion

This paper shows that allowing one production factor—capital—to move across countries can potentially alter the welfare implications of trade liberalization by creating competition across countries for the mobile factor. In the absence of factor mobility, trade liberalization does not change the level of domestic production, so agents enjoy welfare gains purely through the reduction in the domestic price index, as shown by Arkolakis et al. (2012). When capital is mobile, however, attracting additional capital can substantially affect domestic production, especially when firms are heterogeneous and operating requires paying fixed costs.

This paper focuses on tariffs, particularly unilateral tariffs. However, the mechanism is more general: any policy that induces capital inflows can generate similar effects. The central channel is that policies attracting capital inflows encourage additional firm entry by lowering entry costs through cheaper capital. Greater entry expands product variety and shifts consumer expenditure toward domestically produced varieties. As a result, the price-increasing effect of tariffs is dampened, while households benefit from higher wages.

However, the model is purely static and therefore does not incorporate several potentially important economic mechanisms, such as asset markets, price rigidities, or financial market frictions. Accounting for these elements could alter the welfare effects discussed above.

References

Akgul, Z. and Ahmad, S. (2020). Structural parameters of trade models with firm heterogeneity. Conference papers 333222, Purdue University, Center for Global Trade Analysis, Global Trade Analysis Project.

Arkolakis, C., Costinot, A., and Rodríguez-Clare, A. (2012). New trade models, same old gains? *American Economic Review*, 102(1):94–130.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production (une théorie de la demande de produits différenciés d'après leur origine) (una teoría de la demanda de productos distinguiéndolos según el lugar de producción). *Staff Papers (International Monetary Fund)*, 16(1):159–178.

Bagwell, K. and Staiger, R. W. (2012). The economics of trade agreements in the linear cournot delocation model. *Journal of International Economics*, 88(1):32–46.

Boulhol, H. (2009). Do capital market and trade liberalization trigger labor market deregulation? *Journal of International Economics*, 77(2):223–233.

Broda, C. and Weinstein, D. E. (2006). Globalization and the gains from variety*. *The Quarterly Journal of Economics*, 121(2):541–585.

Caliendo, L., Dvorkin, M., and Parro, F. (2019). Trade and labor market dynamics: General equilibrium analysis of the china trade shock. *Econometrica*, 87(3):741–835.

Caliendo, L., Feenstra, R. C., Romalis, J., and Taylor, A. M. (2021). A second-best argument for low optimal tariffs. Working Paper 28380, National Bureau of Economic Research.

Chaney, T. (2008). Distorted gravity: The intensive and extensive margins of international trade. *The American Economic Review*, 98(4):1707–1721.

Costinot, A., Lorenzoni, G., and Werning, I. (2014). A theory of capital controls as dynamic terms-of-trade manipulation. *Journal of Political Economy*, 122(1):77–128.

Edwards, S. and van Wijnbergen, S. (1986). The welfare effects of trade and capital market liberalization. *International Economic Review*, 27(1):141–148.

Head, K. and Mayer, T. (2014). Chapter 3 - gravity equations: Workhorse,toolkit, and cookbook. In Gopinath, G., Helpman, E., and Rogoff, K., editors, *Handbook of International Economics*, volume 4 of *Handbook of International Economics*, pages 131–195. Elsevier.

Helpman, E., Melitz, M. J., and Yeaple, S. R. (2004). Export versus fdi with heterogeneous firms. *The American Economic Review*, 94(1):300–316.

Hu, C. (2024). Portfolio choice analysis in a multi-country macro model. Technical report.

Kleinman, B., Liu, E., Redding, S. J., and Yogo, M. (2023). Neoclassical growth in an interdependent world. Working Paper 31951, National Bureau of Economic Research.

Lloyd, S. P. and Marin, E. A. (2024). Capital controls and trade policy. *Journal of International Economics*, 151:103965.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Melitz, M. J. and Ottaviano, G. I. P. (2008). Market Size, Trade, and Productivity. *The Review of Economic Studies*, 75(1):295–316.

Mundell, R. A. (1957). International trade and factor mobility. *The American Economic Review*, 47(3):321–335.

Ramondo, N. and Rodríguez-Clare, A. (2013). Trade, multinational production, and the gains from openness. *Journal of Political Economy*, 121(2):273–322.

Venables, A. J. (1985). Trade and trade policy with imperfect competition: The case of identical products and free entry. *Journal of International Economics*, 19(1):1–19.

Wong, K.-y. (1986). c. *Journal of International Economics*, 21(1):25–43.

Figures

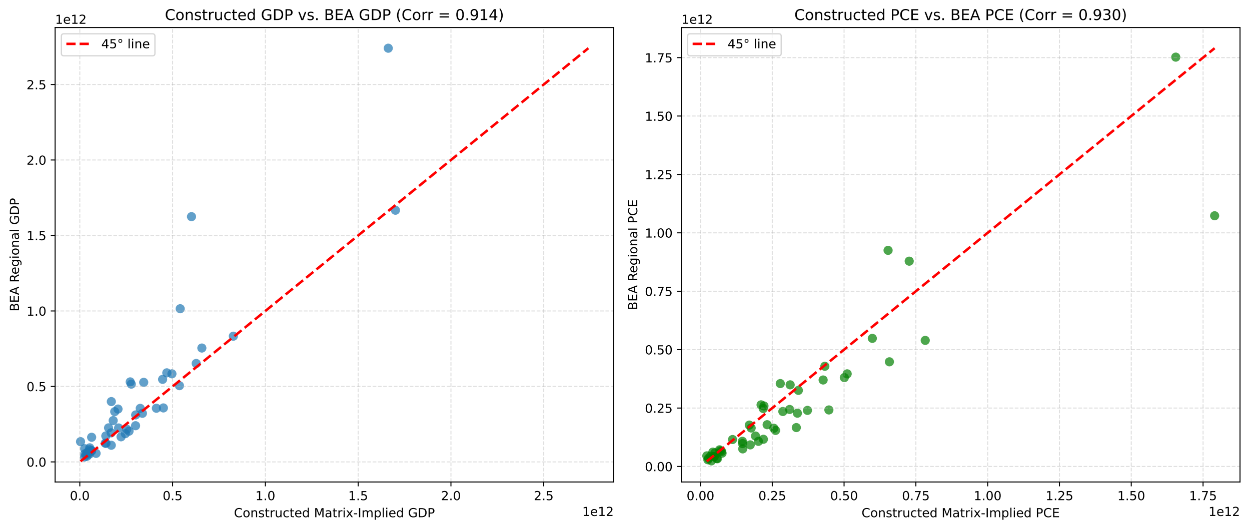

Figure 3: Constructed matrix - implied versus BEA-reported regional values (2017).

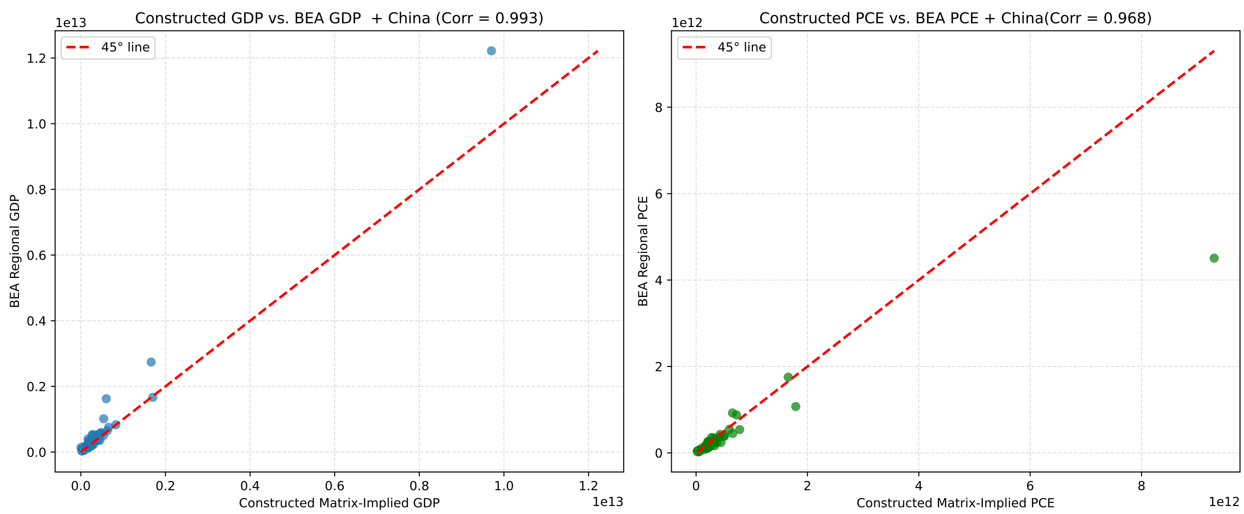

Figure 4: Constructed matrix - implied versus BEA-reported regional values (2017).

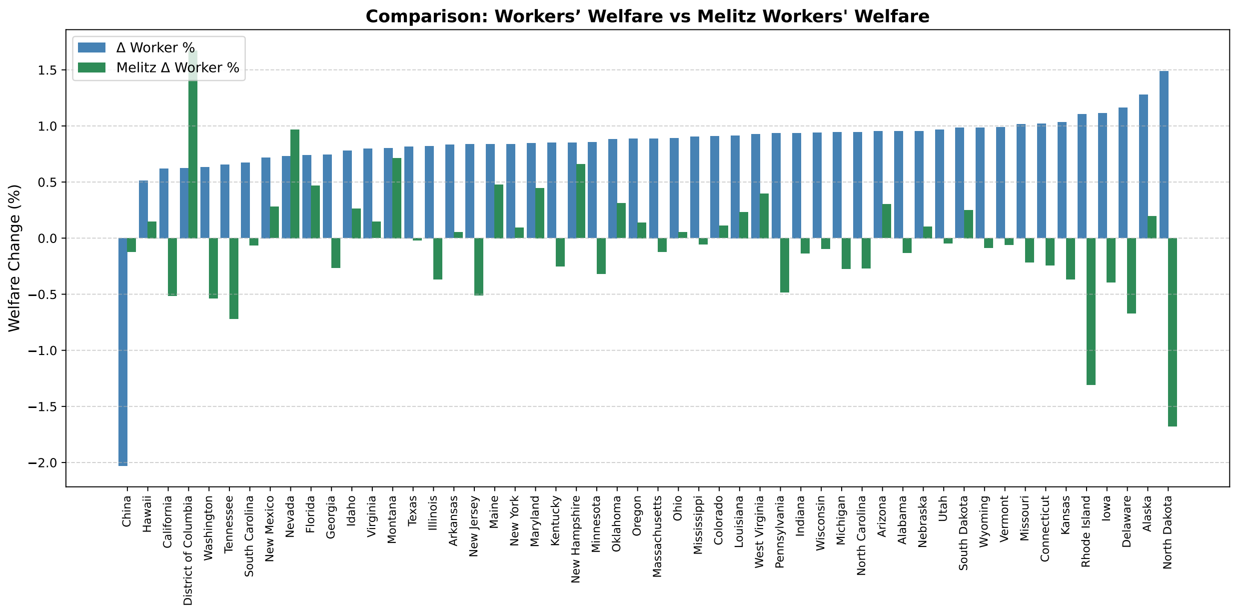

Figure 5: Workers' Welfare Effect Without Retaliation: This Model vs Melitz (2003).

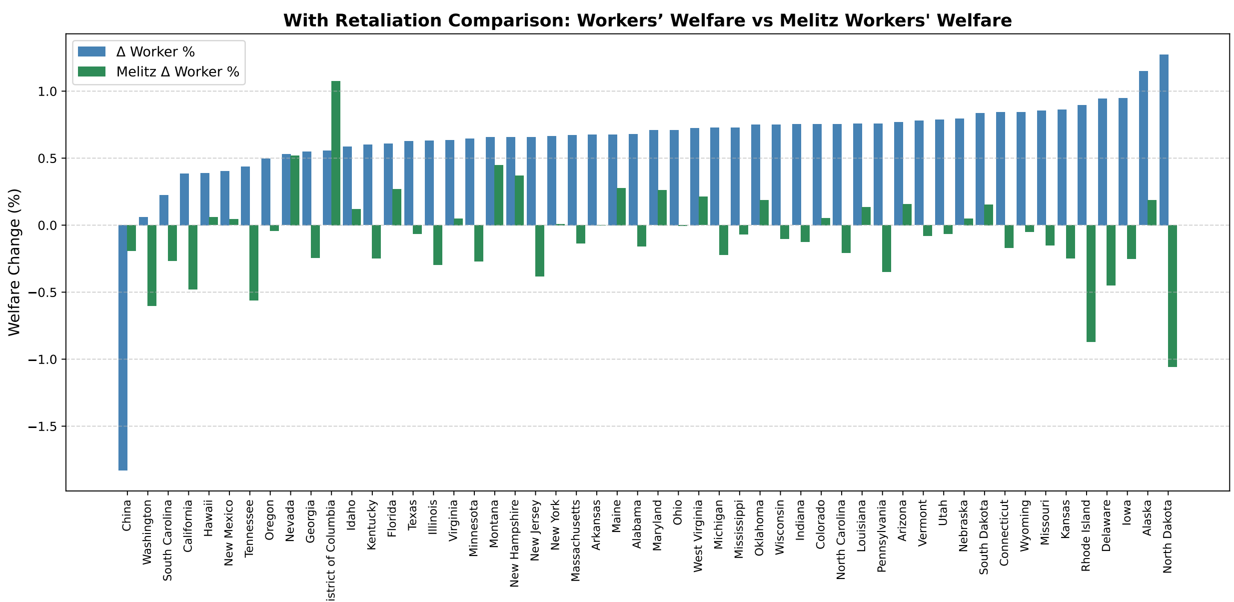

Figure 6: Workers' Welfare Effect With Retaliation: This Model vs Melitz (2003).

Tables

Table 5: (Exclude Washington D.C) Republican Vote Share vs Model Predicted State Welfar Consequences

	(1)	(2)	(3)	(4)	(5)	(6)
Δ HH welfare	6.436*					
	(3.435)					
%Δ HH welfare + retaliation		7.979**				
		(3.717)				
%Δ Worker welfare			18.799**			
			(7.820)			
%Δ Worker welfare + retaliation				16.571***		
				(6.387)		
%Δ Melitz worker welfare					0.449	
					(3.851)	
%Δ Melitz worker welfare + retaliation						2.486
						(5.872)
Constant	0.516***	0.522***	0.325***	0.378***	0.493***	0.495***
	(0.019)	(0.019)	(0.073)	(0.049)	(0.015)	(0.015)
Observations	50	50	50	50	50	50
R-squared	0.064	0.080	0.095	0.109	0.000	0.006
Adj. R-squared	0.044	0.060	0.076	0.091	-0.020	-0.015

Notes: Robust standard errors in parentheses, computed using the HC3 estimator. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

A Proof for symmetric equilibrium solution

Under the totally symmetric assumption, I have

$$\gamma = \frac{1}{2}$$

$$F_J^E = F_L^E = F^E$$

$$F_{JJ} = F_{LL} = F_0$$

$$F_{JL} = F_{LJ} = F_1$$

$$L_J = L_L = L$$

$$\tau_{JL} = \tau_{LJ} = \tau$$

$$\tau_{JJ} = \tau_{LL} = 0$$

Combining the labor market clearing conditions and the free entry conditions gives the expression for the number of firms as a function of wages:

$$N_S = \frac{\sigma - 1}{\sigma\alpha}\frac{W_S^{1-\xi}L}{\xi F^E}. \tag{50}$$

Substituting Equation 50 into the expression for aggregated trade values, Equation 4, gives:

$$V_{SI} = \frac{(\sigma-1)\underline{z}^{\alpha}}{\alpha+1-\sigma}\frac{L}{\xi F^E}W_S^{1-\xi\epsilon}B_I^{\frac{\alpha}{\sigma-1}}F_{SI}^{1+\frac{\alpha}{1-\sigma}}(1+\tau_{SI})^{-\alpha}. \tag{51}$$

Recall that I need to solve for $\{W_J, W_L, B_J, B_L\}$ using two labor market clearing conditions (one for each country), one global capital market clearing condition, and one country's trade balance condition, which are:

$$\sum_I V_{JI} = \frac{W_J L}{\xi},$$

$$\sum_I V_{LI} = \frac{W_L L}{\xi},$$

$$rK_0 = (1-\xi)(\sum_I V_{JI} + \sum_I V_{LI}),$$

$$W_J L + \gamma r K_0 = V_{JJ} + V_{LJ}.$$

The proportional relationship between V_{JJ} and V_{JL} is:

$$\frac{V_{JJ}}{V_{JL}} = \frac{B_J^{\frac{\alpha}{\sigma-1}}F_{JJ}^{1+\frac{\alpha}{1-\sigma}}}{B_L^{\frac{\alpha}{\sigma-1}}F_{JL}^{1+\frac{\alpha}{1-\sigma}}(1+\tau_{JL})^{-\alpha}} = \left(\frac{B_J}{B_L}\right)^{\frac{\alpha}{\sigma-1}}C,$$

where

$$C = \left(\frac{F_0}{F_1}\right)^{1+\frac{\alpha}{1-\sigma}}(1+\tau)^{\alpha}.$$

Similarly,

$$\frac{V_{LL}}{V_{LJ}} = \left(\frac{B_L}{B_J}\right)^{\frac{\alpha}{\sigma-1}}C.$$

Denote

$$\beta = \left(\frac{B_J}{B_L}\right)^{\frac{\alpha}{\sigma-1}}.$$

Then I have

$$V_{JJ} = \beta C V_{JL}, \tag{52}$$

$$V_{LL} = \frac{C}{\beta} V_{LJ}. \tag{53}$$

At the same time,

$$\frac{V_{JL}}{V_{LJ}} = \frac{W_J^{1-\xi\epsilon}}{W_L^{1-\xi\epsilon}} \frac{B_L^{\frac{\alpha}{\sigma-1}} F_{JL}^{1+\frac{\alpha}{1-\sigma}} (1+\tau_{JL})^{-\alpha}}{B_J^{\frac{\alpha}{\sigma-1}} F_{LJ}^{1+\frac{\alpha}{1-\sigma}} (1+\tau_{LJ})^{-\alpha}} = \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon} \left(\frac{B_L}{B_J}\right)^{\frac{\alpha}{\sigma-1}}.$$

So I have

$$\frac{V_{JL}}{V_{LJ}} = \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon} \frac{1}{\beta}. \tag{54}$$

Combining the two labor market clearing conditions gives:

$$\frac{W_J}{W_L} = \frac{V_{JJ} + V_{JL}}{V_{LL} + V_{LJ}} = \frac{V_{JL}(1 + C\beta)}{V_{LJ}(1 + \frac{C}{\beta})}. \tag{55}$$

Substituting Equation 54 into Equation 55 gives:

$$\frac{W_J}{W_L} = \frac{1}{\beta} \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon} \frac{1 + C\beta}{1 + \frac{C}{\beta}}. \tag{56}$$

This further implies

$$\left(\frac{W_J}{W_L}\right)^{\xi\epsilon} = \frac{1 + C\beta}{\beta + C}. \tag{57}$$

Now combining Equation 52 and Equation 53 gives:

$$\frac{V_{JJ}}{V_{LL}} = \frac{\beta C V_{JL}}{\frac{C}{\beta} V_{LJ}} = \beta^2 \frac{V_{JL}}{V_{LJ}} = \beta \left(\frac{W_J}{W_L}\right)^{1-\xi\epsilon}. \tag{58}$$

Substituting Equation 57 back gives:

$$\frac{V_{JJ}}{V_{JL}} = \frac{W_J}{W_L} \frac{\beta^2 + C\beta}{1 + C\beta}, \tag{59}$$

and

$$\frac{V_{JL}}{V_{LJ}} = \frac{W_J}{W_L} \frac{\beta + C}{\beta + C\beta^2}. \tag{60}$$

Recall that combining the labor market clearing conditions gives:

$$\frac{W_J}{W_L} = \frac{V_{JJ} + V_{JL}}{V_{LL} + V_{LJ}}.$$

Representing V_{JJ} using V_{LL} and V_{JL} using V_{LJ} gives:

$$\frac{W_J}{W_L} = \frac{\frac{W_J}{W_L}\frac{\beta^2+C\beta}{1+C\beta}V_{LL} + \frac{W_J}{W_L}\frac{\beta+C}{\beta+C\beta^2}V_{LJ}}{V_{LL}+V_{LJ}}.$$

This further implies

$$1 = \frac{\frac{\beta^2+C\beta}{1+C\beta}V_{LL} + \frac{\beta+C}{\beta+C\beta^2}V_{LJ}}{V_{LL}+V_{LJ}}. \tag{61}$$

For this equation to hold, it must be that

$$\frac{\beta^2+C\beta}{1+C\beta} = \frac{\beta+C}{\beta+C\beta^2} = 1, \tag{62}$$

which implies $\beta = 1$, i.e. $B_J = B_L$. From Equation 57, this also implies $W_J = W_L$.

Using $B_J = B_L = B$ and $W_J = W_L = W$, I can solve for

$$W = \frac{L\xi}{2K_0(1-\xi)}$$

and

$$B^{\frac{\alpha}{\sigma-1}} = \left(\frac{L\xi}{2K_0(1-\xi)}\right)^{\xi\epsilon} \frac{\alpha+1-\sigma}{(\sigma-1)\underline{z}^{\alpha}} \frac{F^E}{F_0^{1+\frac{\alpha}{1-\sigma}} + F_1^{1+\frac{\alpha}{1-\sigma}}(1+\tau)^{-\alpha}}.$$

Therefore, the solution of the model under symmetric equilibrium is as follows:

$$\begin{pmatrix} W_J \\ W_L \\ B_J \\ B_L \end{pmatrix} = \begin{pmatrix} \frac{L\xi}{2K_0(1-\xi)} \\ \frac{L\xi}{2K_0(1-\xi)} \\ \left[\left(\frac{L\xi}{2K_0(1-\xi)}\right)^{\xi\epsilon} \frac{\alpha+1-\sigma}{(\sigma-1)\underline{z}^{\alpha}} \frac{F^E}{F_0^{1+\frac{\alpha}{1-\sigma}} + F_1^{1+\frac{\alpha}{1-\sigma}}(1+\tau)^{-\alpha}}\right]^{\frac{\sigma-1}{\alpha}} \\ \left[\left(\frac{L\xi}{2K_0(1-\xi)}\right)^{\xi\epsilon} \frac{\alpha+1-\sigma}{(\sigma-1)\underline{z}^{\alpha}} \frac{F^E}{F_0^{1+\frac{\alpha}{1-\sigma}} + F_1^{1+\frac{\alpha}{1-\sigma}}(1+\tau)^{-\alpha}}\right]^{\frac{\sigma-1}{\alpha}} \end{pmatrix}.$$

B Immobile Capital

This appendix considers a variant of the model in Section 2 in which capital is immobile across countries. I show that, under immobile capital, the equilibrium mass of firms is pinned down by exogenous parameters rather than adjusting to endogenous equilibrium variables.

As in Section 2, the total world capital endowment is K_0. Households in country J own a share γ of this endowment, while households in country L own the remaining share $1-\gamma$.

The parameters K_0, γ, and $1-\gamma$ are all exogenously given.

B.1 Consumer Side

Consumers (households) in destination country $I \in \{J, L\}$ solve the same utility-maximization problem subject to their budget constraint as in Section 2, except that the rental rate of capital is now country specific. In particular, instead of a common rental rate r, consumers in country I face r_I.

Given this modification, the demand for variety ω by consumers in country I is

$$q_I(\omega) = \frac{p_I(\omega)^{-\sigma}}{P_I^{1-\sigma}} \left(r_I K_I + W_I L_I \right).$$

B.2 Firm Side

On the firm side, the only modification relative to Section 2 is that the rental rate of capital is country specific. Firms located in country $S \in \{J, L\}$ therefore face the rental rate r_S.

As illustrated in Section 2, with constant source–destination iceberg trade costs, there is a one-to-one mapping between the quantity consumed in destination I, $q_I(\omega)$, and the quantity shipped by a firm with productivity z from source S to destination I, $q_{SI}(z)$.

Given demand for goods shipped to destination I, a firm located in country S requires the following inputs:

$$K_{SI}(z) = (1-\xi)\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r_S}\right)^{\xi}(1+\tau_{SI}),$$

$$L_{SI}(z) = \xi\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r_S}\right)^{\xi-1}(1+\tau_{SI}).$$

Finally, under monopolistic competition, firms set prices as a constant markup over marginal cost:

$$p_{SI}(z) = \frac{\sigma}{\sigma-1}\frac{W_S^{\xi} r_S^{1-\xi}}{z}(1+\tau_{SI}).$$

B.3 Aggregated Trade Values

The trade value generated by a firm with productivity z located in country S and selling to destination I is

$$V_{SI}(z) = \sigma B_I \left(\frac{W_S^{\xi} r_S^{1-\xi}}{z}(1+\tau_{SI})\right)^{1-\sigma}.$$

Aggregating over all firms in country S that export to market I, total trade values are given by

$$V_{SI} = \frac{\sigma\alpha\underline{z}^{\alpha}}{\alpha+1-\sigma} N_S B_I^{\frac{\alpha}{\sigma-1}} F_{SI}^{1+\frac{\alpha}{1-\sigma}} \left(W_S^{\xi} r_S^{1-\xi}\right)^{1-\epsilon} (1+\tau_{SI})^{-\alpha}. \tag{63}$$

Relative to Equation 4, the only difference is that the rental rate of capital is now country specific.

B.4 Technology Threshold for Selling in a Market

The productivity threshold for a firm located in country S to profitably sell in destination market I is analogous to Equation 3, except that the rental rate of capital is now country specific. Specifically,

$$z_{\min}^{SI} = \left(\frac{F_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}} \left(W_S^{\xi} r_S^{1-\xi}\right)^{\frac{\sigma}{\sigma-1}} (1+\tau_{SI}).$$

B.5 Equilibrium Conditions

With immobile capital, the global capital-market–clearing condition is replaced by country-specific capital-market–clearing conditions, each associated with its own rental rate. Normalizing the wage in country J to one, the endogenous variables are

$$W_L,\ r_J,\ r_L,\ N_J,\ N_L,\ B_J,\ B_L.$$

These seven endogenous variables are pinned down by seven equilibrium conditions. For each country, these include:

1. a free-entry condition,
2. a labor-market–clearing condition, and
3. a capital-market–clearing condition.

In addition, one trade-balance condition closes the model.

The free-entry conditions are

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{JI} = N_J F_J^E W_J^{\xi} r_J^{1-\xi}, \tag{64}$$

$$\frac{\sigma-1}{\sigma\alpha}\sum_I V_{LI} = N_L F_L^E W_L^{\xi} r_L^{1-\xi}. \tag{65}$$

Labor-market–clearing conditions are

$$W_J L_J = \xi \sum_I V_{JI}, \tag{66}$$

$$W_L L_L = \xi \sum_I V_{LI}. \tag{67}$$

Capital-market–clearing conditions are

$$r_J K_J = r_J \gamma K_0 = (1 - \xi) \sum_I V_{JI}, \tag{68}$$

$$r_L K_L = r_L (1 - \gamma) K_0 = (1 - \xi) \sum_I V_{LI}. \tag{69}$$

Finally, the trade-balance condition is

$$V_{JL} = V_{LJ}. \tag{70}$$

Notice that combining the country-specific labor-market–clearing condition with the capital-market–clearing condition implies

$$\frac{W_S}{r_S} = \frac{\xi}{1 - \xi} \frac{K_S}{L_S}. \tag{71}$$

Equation (71) shows that the relative factor price is pinned down by factor endowments, which are exogenous. As a result, the rental rate r_S can be expressed as a function of the wage W_S, allowing r_S to be eliminated as an independent equilibrium object. The set of endogenous variables can therefore be reduced to

$$\{W_L,\, N_J,\, N_L,\, B_J,\, B_L\}.$$

Next, combining the country-specific free-entry condition with the labor-market–clearing condition yields

$$N_S = \frac{\sigma - 1}{\sigma \alpha} \frac{L_S}{\xi F_S^E} \left(\frac{W_S}{r_S} \right)^{1-\xi}. \tag{72}$$

Substituting Equation (71) into Equation (72) gives

$$N_S = \frac{\sigma - 1}{\sigma \alpha} \frac{1}{\xi F_S^E} \left(\frac{\xi}{1 - \xi} \right)^{1-\xi} K_S^{1-\xi} L_S^{\xi}. \tag{73}$$

Equation (73) implies that the equilibrium mass of firms N_S depends solely on exogenous factor endowments and structural parameters. Consequently, N_J and N_L can also be eliminated as equilibrium objects, further reducing the set of endogenous variables to

$$\{W_L,\, B_J,\, B_L\}.$$

Equivalently, after substituting out r_S and N_S, the equilibrium system is isomorphic to one in which labor is the only immobile input, with capital affecting equilibrium outcomes only through the composite term $K_S^{1-\xi} L_S^{\xi}$.

As a result, to determine the remaining endogenous variables $\{W_L, B_J, B_L\}$, it suffices to impose the two country-specific labor-market–clearing conditions together with the trade-balance condition:

$$W_J L_J = \xi \sum_I V_{JI}, \tag{74}$$

$$W_L L_L = \xi \sum_I V_{LI}, \tag{75}$$

$$V_{JL} = V_{LJ}. \tag{76}$$

Number of firms From Equation (72), all variables on the right-hand side are exogenous. In particular, K_S and L_S are fixed country-specific factor endowments, while ξ and F_S^E are structural parameters. Therefore, under immobile capital, the equilibrium mass of firms in country $S \in \{J, L\}$ is pinned down entirely by exogenous primitives and does not respond to endogenous equilibrium outcomes such as wages, rental rates, or market size. This result highlights that it is the interaction between capital usage in production and capital mobility that causes firm entry to respond endogenously to factor prices, as shown in Section 2.

B.6 Comparative Statics

Log-differentiate Equation 63 gives:

$$\hat{V}_{SI} = \frac{\alpha}{\sigma - 1}\hat{B}_I + (1-\epsilon)\xi\hat{W}_S + (1-\xi)(1-\epsilon)\hat{r}_S - \alpha\hat{\tau}_{SI}$$

Since $\frac{W_S}{r_S}$ is equal to a constant, I have

$$\hat{W}_S = \hat{r}_S$$

So

$$\hat{V}_{SI} = \frac{\alpha}{\sigma - 1}\hat{B}_I + (1-\epsilon)\hat{W}_S - \alpha\hat{\tau}_{SI} \tag{77}$$

Given this, I can then log-differentiate equilibrium conditions: For country J's labor market:

$$\sum_I V_{SI}\hat{W}_S = \sum_I V_{SI}\hat{V}_{SI}$$

Denote $\lambda_S = \frac{V_{SI}}{\sum_I V_{SI}}$, under symmetry, $\lambda_J = \lambda_L = \lambda$ So I have:

$$\hat{W}_J = \lambda\hat{V}_{JL} + (1-\lambda)\hat{V}_{JJ} \tag{78}$$

Sicne W_J is the numeraire, this furtherly becomes:

$$\lambda\hat{V}_{JL} = (\lambda - 1)\hat{V}_{JJ} \tag{79}$$

For country L:

$$\hat{W}_L = \lambda\hat{V}_{LJ} + (1-\lambda)\hat{V}_{LL} \tag{80}$$

Finally for the trade balance condition

$$\hat{V}_{JL} = \hat{V}_{LJ} \tag{81}$$

Combining them together, the system can be written in matrix form as

$$\begin{pmatrix} 0 & -\dfrac{\alpha(1-\lambda)}{\sigma-1} & -\dfrac{\alpha\lambda}{\sigma-1} \\ \epsilon & -\dfrac{\alpha\lambda}{\sigma-1} & -\dfrac{\alpha(1-\lambda)}{\sigma-1} \\ 1-\epsilon & \dfrac{\alpha}{\sigma-1} & \dfrac{\alpha}{\sigma-1} \end{pmatrix} \begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} -\alpha\lambda\,\hat{\tau}_{JL} \\ -\alpha\lambda\,\hat{\tau}_{LJ} \\ \alpha(\hat{\tau}_{LJ} - \hat{\tau}_{JL}) \end{pmatrix}.$$

Bilateral trade liberalization Suppose trade costs fall symmetrically,

$$\hat{\tau}_{JL} = \hat{\tau}_{LJ} = \hat{\tau} < 0.$$

Solving the system yields

$$\begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda(\sigma-1)\hat{\tau} \\ \lambda(\sigma-1)\hat{\tau} \end{pmatrix}.$$

Using the relationship $-\hat{P}_I = -\hat{B}_I/(\sigma-1)$, both countries experience identical welfare gains equal to

$$-\lambda\hat{\tau}.$$

Hence, the welfare gains from bilateral trade liberalization coincide with those obtained in Section 2.

Unilateral trade liberalization Now consider a unilateral reduction in trade costs from country L to country J,

$$\hat{\tau}_{LJ} = \hat{\tau} < 0, \qquad \hat{\tau}_{JL} = 0.$$

The solution to the system is

$$\begin{pmatrix} \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} -\dfrac{\alpha(1-\lambda)\hat{\tau}}{A} \\ (\sigma-1)\dfrac{\lambda B\,\hat{\tau}}{A} \\ -(\sigma-1)\dfrac{(1-\lambda)B\,\hat{\tau}}{A} \end{pmatrix},$$

where $A \equiv (\epsilon-1)(1-2\lambda)$ and $B \equiv \epsilon(1-\lambda)+\lambda$.

From Equation (78), it is immediate that in this environment trade-cost changes do not affect domestic GDP. In contrast, as shown in Section 2, Paragraph 2.9.4, a unilateral tariff change does alter domestic GDP when capital is mobile. This difference highlights the role of capital mobility in transmitting trade-policy shocks to aggregate production.

The domestic price index change will be simply

$$\hat{P}_J = \frac{1}{\sigma-1}\hat{B}_J \tag{82}$$

where substituting the solution gives

$$\hat{P}_J = -\frac{\lambda}{1-\lambda}\hat{B}_L \tag{83}$$

C Different Input Factor Intensity for Fixed Costs

Suppose that the factor intensity of fixed costs differs from that of variable production. Specifically, firms produce final goods using the Cobb–Douglas technology

$$z\left(\frac{l}{\xi}\right)^{\xi}\left(\frac{k}{1-\xi}\right)^{1-\xi},$$

where $\xi \in (0,1)$ denotes the labor share in variable production.

In contrast, fixed costs are incurred using a different factor intensity. To serve destination

I, firms located in country S must incur a fixed cost that uses labor and capital according to

$$\frac{1}{F_{SI}}\left(\frac{l}{\phi}\right)^{\phi}\left(\frac{k}{1-\phi}\right)^{1-\phi},$$

where $\phi \in (0,1)$ denotes the labor share in fixed costs. As a result, the fixed cost of serving market I for firms located in country S is given by

$$F_{SI}W_S^{\phi}r^{1-\phi}.$$

Similarly, the fixed cost of entry for firms located in country S is given by

$$F_S^E W_S^{\phi}r^{1-\phi}.$$

C.1 Consumer Side, Firm Side, and Trade Values

Allowing fixed costs to have a different factor intensity does not affect variable-cost–related calculations. As a result, consumer demand, firm pricing, and factor demands associated with variable production remain unchanged.

In particular, the quantity demanded of a variety with productivity z shipped from country S to country I is

$$q_{SI}(z) = \frac{\sigma}{\sigma-1}\frac{W_S^{\xi}r^{1-\xi}}{z}(1+\tau_{SI}).$$

The firm's factor demands for variable production are therefore unchanged and given by

$$K_{SI}(z) = (1-\xi)\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r}\right)^{\xi}(1+\tau_{SI}),$$

$$L_{SI}(z) = \xi\frac{q_{SI}(z)}{z}\left(\frac{W_S}{r}\right)^{\xi-1}(1+\tau_{SI}).$$

The optimal price charged by a firm from country S selling to country I is

$$p_{SI}(z) = \frac{\sigma}{\sigma-1}\frac{W_S^{\xi}r^{1-\xi}}{z}(1+\tau_{SI}).$$

Finally, revenue earned by a firm with productivity draw z selling from S to I is given by

$$V_{SI}(z) = \sigma B_I\left(W_S^{\xi}r^{1-\xi}(1+\tau_{SI})\right)^{1-\sigma}z^{\sigma-1}.$$

All expressions above are identical to those in Section 2.

C.2 Technology Threshold and Aggregate Trade Value

The first deviation from Section 2 concerns the productivity threshold for firms located in country S to serve market I. With a different factor intensity for fixed costs, the zero-profit cutoff condition is now given by

$$\tilde{\pi}_{SI}\left(z_{SI}^{\min}\right) = F_{SI} W_S^{\phi} r^{1-\phi}.$$

This condition implies the following productivity threshold:

$$z_{SI}^{\min} = \left(\frac{F_{SI}}{B_I}\right)^{\frac{1}{\sigma-1}} W_S^{\frac{\phi}{\sigma-1}+\xi} r^{\frac{1-\phi}{\sigma-1}+1-\xi}(1+\tau_{SI}).$$

Aggregate trade flows from country S to market I are therefore given by

$$V_{SI} = \frac{\sigma \alpha_S \underline{z}_S^{\alpha_S}}{\alpha_S + 1 - \sigma} N_S \, F_{SI}^{1+\frac{\alpha_S}{1-\sigma}} \, B_I^{\frac{\alpha_S}{\sigma-1}} \, (1+\tau_{SI})^{-\alpha_S} \, W_S^{\phi - \frac{\alpha_S \phi}{\sigma-1} - \alpha_S \xi} \, r^{1-\phi-\frac{\alpha_S(1-\phi)}{\sigma-1} - \alpha_S(1-\xi)}.$$

C.3 CRS Properties for Factor Prices and Market-Clearing Conditions

We first verify that the cost function remains homogeneous of degree one in factor prices when fixed costs have a different factor intensity. The total cost incurred by a firm from country S selling to market I can be written as

$$C(q, W_S, r) = F_{SI} W_S^{\phi} r^{1-\phi} + F_S^E W_S^{\phi} r^{1-\phi} + W_S^{\xi} r^{1-\xi} \frac{q_{SI}(z)(1+\tau_{SI})}{z}.$$

It follows immediately that

$$C(q, \lambda W_S, \lambda r) = \lambda C(q, W_S, r),$$

so the cost function remains homogeneous of degree one in factor prices (W_S, r). Equilibrium allocations therefore depend only on relative factor prices.

Turning to market-clearing conditions, labor is used in variable production as well as in the payment of fixed costs for market access and entry. Under the symmetric-country assumption, total labor used by firms from country S to serve destination I can be written as

$$\tilde{L}_{SI} = \frac{\xi(\sigma-1)}{\sigma W_S} V_{SI} + \frac{\phi(\alpha+1-\sigma)}{\sigma \alpha W_S} V_{SI}.$$

Aggregating across destinations and adding entry fixed costs yields total labor demand in country S:

$$L_S = \left[\frac{(\sigma-1)\xi + \phi}{\sigma W_S}\right] \sum_I V_{SI}.$$

Hence, labor-market–clearing conditions can be written compactly as

$$L_J = \frac{(\sigma-1)\xi + \phi}{\sigma W_J} \sum_I V_{JI}, \qquad L_L = \frac{(\sigma-1)\xi + \phi}{\sigma W_L} \sum_I V_{LI}.$$

Similarly, capital is used in variable production and in the payment of fixed costs. The global capital-market–clearing condition is therefore given by

$$K_0 = \frac{(\sigma-1)(1-\xi) + (1-\phi)}{\sigma r} \sum_S \sum_I V_{SI}.$$

Note that when $\phi = \xi$, the market-clearing conditions collapse to those in Section 2.

C.4 Firm Entry Decisions

Combining the free-entry condition

$$\frac{\sigma-1}{\sigma\alpha} \sum_I V_{SI} = N_S F_S^E W_S^{\phi} r^{1-\phi}$$

with the labor-market–clearing condition

$$\sigma W_S L_S = ((\sigma-1)\xi + \phi) \sum_I V_{SI}$$

yields the equilibrium mass of firms:

$$N_S = \frac{(\sigma-1) L_S}{\alpha F_S^E \left((\sigma-1)\xi + \phi\right)} \left(\frac{W_S}{r}\right)^{1-\phi}.$$

As in the baseline model, firm entry depends on relative factor prices. A higher rental–wage ratio raises the cost of entry thus reducing the equilibrium mass of firms. Allowing fixed costs to have a different factor intensity alters the elasticity of entry with respect to factor prices but does not change the underlying mechanism.

D Constant Fixed Costs

This appendix considers the case in which the fixed cost of serving market I from country S, denoted f_{SI}, and the fixed cost of operating (entry), denoted f_S^E, are both constants (i.e., they do not scale with factor prices). Since the rental rate of capital is chosen as the numeraire, these fixed costs are denominated in units of capital and therefore do not vary with equilibrium wages. Equivalently, this specification corresponds to setting the wage intensity parameter ϕ introduced in Appendix C equal to zero. This modification does not affect the household problem or firms' pricing rules, but it alters the export cutoff and the free-entry condition.

Revenue at the firm level. For a firm in S with productivity z selling to market I, revenue is unchanged:

$$V_{SI}(z) = \sigma B_I \Big(W_S^{\xi} r^{1-\xi}(1+\tau_{SI}) \Big)^{1-\sigma} z^{\sigma-1}.$$

Market-access cutoff. Let $\pi_{SI}(z)$ denote operating profits (before paying the market-specific fixed cost). With CES demand and monopolistic competition,

$$\pi_{SI}(z) = \frac{1}{\sigma} V_{SI}(z).$$

The cutoff productivity $z_{\min}^{SI}$ is pinned down by the zero-profit condition

$$\pi_{SI}(z_{\min}^{SI}) = f_{SI},$$

which implies

$$z_{\min}^{SI} = \left(\frac{\sigma f_{SI}}{B_I} \right)^{\frac{1}{\sigma-1}} W_S^{\xi} r^{1-\xi}(1+\tau_{SI}).$$

Aggregate trade value. The aggregate value of sales from S to I is

$$V_{SI} = N_S \int_{z_{\min}^{SI}}^{\infty} V_{SI}(z) f^S(z)\, dz = \frac{\sigma \alpha_S}{\alpha_S + 1 - \sigma} N_S\, \underline{z}_S^{\alpha_S}\, B_I \Big(W_S^{\xi} r^{1-\xi}(1+\tau_{SI}) \Big)^{1-\sigma} \left(z_{\min}^{SI} \right)^{\sigma-1-\alpha_S}.$$

Substituting the cutoff yields

$$V_{SI} = N_S \frac{\sigma \alpha_S \underline{z}^{\alpha_S}}{\alpha_S + 1 - \sigma} f_{SI}^{1+\frac{\alpha_S}{1-\sigma}} B_I^{\frac{\alpha_S}{\sigma-1}} (\Big(W_S^{\xi} r^{1-\xi}(1+\tau_{SI}) \Big)^{-\alpha_S}$$

Profits net of market-access fixed costs. Profits net of the market-specific fixed cost are

$$\tilde{\pi}_{SI}(z) = \pi_{SI}(z) - f_{SI} = \frac{V_{SI}(z)}{\sigma} - f_{SI}.$$

Aggregating across active firms,

$$\tilde{\pi}_{SI} = N_S \int_{z_{\min}^{SI}}^{\infty} \tilde{\pi}_{SI}(z) f^S(z)\, dz = \frac{\sigma - 1}{\sigma \alpha_S} V_{SI},$$

where the last equality uses the Pareto assumption and the cutoff defined by $\pi_{SI}(z_{\min}^{SI}) = f_{SI}$.

Free-entry condition. Let total expected operating profits net of market-specific fixed costs be $\sum_I \tilde{\pi}_{SI}$. With constant entry cost f_S^E, the free-entry condition is

$$\sum_I \tilde{\pi}_{SI} = N_S f_S^E, \qquad \text{equivalently} \qquad \sum_I \frac{\sigma - 1}{\sigma \alpha_S} V_{SI} = N_S f_S^E.$$

Labor market clearing. Labor is used only in variable production. Variable labor demand for a firm with productivity z serving I is proportional to revenue:

$$W_S L_{SI}(z) = \xi \frac{\sigma - 1}{\sigma} V_{SI}(z).$$

Aggregating across destinations and active firms yields

$$W_S L_S = \sum_I N_S \int_{z_{\min}^{SI}}^{\infty} W_S L_{SI}(z) f^S(z)\, dz = \xi \frac{\sigma - 1}{\sigma} \sum_I V_{SI}.$$

Combining labor market clearing with the free-entry condition gives an expression for the equilibrium mass of entrants:

$$N_S = \frac{\sigma - 1}{\sigma \alpha_S} \frac{\sum_I V_{SI}}{f_S^E} = \frac{W_S L_S}{\xi \alpha_S f_S^E}.$$

Takeaway. This appendix highlights a general implication of combining heterogeneous-firm free entry with an integrated capital market. Regardless of whether fixed costs are specified in units of factors (e.g., $W_S^{\xi} r^{1-\xi}$) or as constants, the equilibrium mass of entrants N_S is no longer pinned down solely by endowments and parameters.

E Analysis of the elasticity

In an Armington-style model, regardless of capital mobility, the elasticity is

$$1 - \sigma < 0.$$

Chaney (2008) shows that within the Melitz (2003) framework, assuming a Pareto distribution of firm productivity and no firm entry or exit, the elasticity takes the form

$$-\alpha < 0.$$

If firm entry and exit are allowed within the Chaney (2008) framework, the elasticity becomes

$$1 - \epsilon < 0.$$

How the difference arises To clarify the source of these differences, note first that the price index is a function of domestic and foreign unit factor prices. With the rental rate r chosen as the numeraire, unit factor prices depend solely on country-specific wages. Consequently, all equilibrium adjustments are ultimately driven by wage changes.

However, the log-differential of the price index is not linear in the log-differentials of wages. I therefore separate the effect of wage changes from the effect operating through the price index. When discussing the influence of wages on trade values, I refer to the *direct* effect of wages holding the price index fixed, as opposed to the *indirect* effects transmitted through changes in the price index (or, equivalently, B_I).

To build intuition, I begin at the firm level. Consider a firm whose productivity z is sufficiently high that its market participation decisions are unaffected by a potential tariff change. Log-differentiating Equation 1 yields

$$\hat{V}_{SI}(z) = \xi(1-\sigma)\hat{W}_S + (1-\xi)(1-\sigma)\hat{r} + \hat{B}_I + (1-\sigma)\hat{\tau}_{SI}. \tag{84}$$

When capital is immobile, the rental rate is country-specific. As shown in Appendix B, relative factor prices are pinned down by fixed factor endowments, implying $\hat{r}_S = \hat{W}_S$. Substituting this condition yields

$$\hat{V}_{SI}(z) = (1-\sigma)\hat{W}_S + \hat{B}_I + (1-\sigma)\hat{\tau}_{SI}. \tag{85}$$

Holding wages fixed, firm-level trade values admit a partial-equilibrium interpretation. Demand in market I is downward sloping, while supply is perfectly elastic at the constant-markup price implied by marginal cost. Firm-level sales are therefore determined by the intersection of a downward-sloping demand curve and a horizontal supply curve. Trade values reflect the resulting trade-off between prices and quantities as wages change.

In this case, the elasticity of firm-level sales with respect to wages is $1-\sigma$, exactly as in the Armington (1969) model. In the absence of heterogeneous-firm entry and exit, this firm can be interpreted as a representative firm for the country, so its sales elasticity coincides with the aggregate elasticity, replicating the Armington result.

If we go one step further and allow for heterogeneous firms, the relevant object becomes aggregate trade values. Log-differentiating Equation 4 yields

$$\hat{V}_{SI} = \hat{N}_S + \hat{B}_I + \xi(1-\sigma)\hat{W}_S + (1-\xi)(1-\sigma)\hat{r} + (1-\sigma)\hat{\tau}_{SI} + (\sigma-\alpha-1)\hat{Z}^{SI}_{\min}. \tag{86}$$

Under the assumption that the fixed cost is a constant instead of responding to the factor price, as shown in Chaney (2008), this implies

$$\hat{Z}^{SI}_{\min} = \xi\hat{W}_S + (1-\xi)\hat{r} + \frac{1}{1-\sigma}\hat{B}_I + \hat{\tau}_{SI}. \tag{87}$$

Factors are immobile so that $\hat{r}_S = \hat{W}_S$, and manually fixing the entry of firms (as Chaney (2008) did), aggregate trade values reduce to

$$\hat{V}_{SI} = -\alpha\hat{W}_S + \frac{\alpha}{\sigma-1}\hat{B}_I - \alpha\hat{\tau}_{SI}, \tag{88}$$

which replicates the elasticity result in Chaney (2008).

If we further allow for free entry and exit, and let the fixed cost respond to factor prices, then

$$\hat{Z}^{SI}_{\min} = \frac{1}{1-\sigma}\hat{B}_I + \frac{\sigma}{\sigma-1}\big(\xi\hat{W}_S + (1-\xi)\hat{r}\big) + \hat{\tau}_{SI}. \tag{89}$$

Substituting Equation (89) into Equation (86) yields

$$\hat{V}_{SI} = \hat{N}_S + \frac{\alpha}{1-\sigma}\hat{B}_I + \xi\left(1-\frac{\sigma\alpha}{\sigma-1}\right)\hat{W}_S + (1-\xi)\left(1-\frac{\sigma\alpha}{\sigma-1}\right)\hat{r} - \alpha\hat{\tau}_{SI}. \tag{90}$$

Denote $\frac{\sigma\alpha}{\sigma-1} \equiv \epsilon$. Since $\hat{N}_S = 0$, the wage elasticity is $\xi(1-\epsilon)$.

Finally, if capital is immobile, then $\hat{N}_S = (1-\xi)\hat{W}_S$, and the elasticity becomes

$$(1-\xi) + \xi - \xi\epsilon = 1 - \xi\epsilon,$$

reflecting the combined intensive- and extensive-margin responses.

F Matrix Form with $\hat{N}_J$ and $\hat{N}_L$

$$\begin{pmatrix} 1 & 0 & \xi-1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \xi-1 & 0 & 0 \\ \sum_I V_{JI} & 0 & (1-\xi(1-\epsilon))\sum_I V_{JI} & 0 & \frac{\alpha}{1-\sigma}V_{JJ} & \frac{\alpha}{1-\sigma}V_{JL} \\ 0 & \sum_I V_{LI} & 0 & (1-\xi(1-\epsilon))\sum_I V_{LI} & \frac{\alpha}{1-\sigma}V_{LJ} & \frac{\alpha}{1-\sigma}V_{LL} \\ 0 & 0 & \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\ -V_{JJ} & -V_{LJ} & \xi\sum_I V_{JI} - \xi(1-\epsilon)V_{JJ} & -\xi(1-\epsilon)V_{LJ} & \frac{\alpha}{1-\sigma}(V_{JJ}+V_{LJ}) & 0 \end{pmatrix} \begin{pmatrix} \hat{N}_J \\ \hat{N}_L \\ \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -\alpha V_{JL}\hat{\tau}_{JL} \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -\alpha V_{LJ}\hat{\tau}_{LJ} \end{pmatrix}$$

G Term $A > 0$

First, we evaluate the sign of A. It is reasonable to assume that the fixed cost for exporting is bigger than fixed cost for selling in the domestic market, and the iceberg cost will never be negative. Given these assumption, I have $\lambda < \frac{1}{2}$, i.e, the proportion of GDP from exporting is always less than 50%. Then I have $1-\lambda > \frac{1}{2}$. So $A > (1-\xi) + 2\lambda\xi + 2\lambda(\epsilon\xi - 1) = (1-\xi) + 2\lambda\xi + 2\lambda\epsilon\xi - 2\lambda = (1-\xi) + 2\lambda(\xi-1) + 2\lambda\epsilon\xi = (1-\xi)(1-2\lambda) + 2\lambda\epsilon\xi > 0$.

H Order of conditions

I will list these conditions again here:

First the condition for price index to increase with a unilateral trade liberalization, thus make investors lose welfare from the unilateral trade liberalization:

$$\lambda((1-2\lambda)(1-\epsilon\xi) - \xi) > \epsilon\xi \tag{91}$$

Then the condition for workers to lose welfare from the unilateral trade liberalization:

$$(1 + \frac{\xi}{\sigma - 1})\alpha(1-\lambda) + \lambda((1-2\lambda)(1-\epsilon\xi) - \xi) > \epsilon\xi \tag{92}$$

Finally the condition for export share of GDP to decrease with a unilateral trade liberalization:

$$(1 - 2\lambda(1 - \epsilon\xi)) - \xi > \epsilon\xi \tag{93}$$

Given the common term one can see immediately that Condition 91 is stricter than Condition 92. Additionally, assume $\lambda < \frac{1}{2}$, it is also immediate that Condition 91 is stricter than Condition 93.

To decide the strictness between Condition 93 and Condition 92, I firstly assume that Condition 93 is satisfied, then I will prove that Condition 92 is automatically satisfied after.

Assume Condition 93 is satisfied gives:

$$\lambda((1 - 2\lambda)(1 - \epsilon\xi) - \xi) > \lambda\epsilon\xi$$

Substitute the above term back to Condition 92, I get:

$$(1 + \frac{\xi}{\sigma - 1})\alpha(1 - \lambda) + \lambda((1 - 2\lambda)(1 - \epsilon\xi) - \xi) > (1 + \frac{\xi}{\sigma - 1})\alpha(1 - \lambda) + \lambda\epsilon\xi$$

If I can prove that

$$(1 + \frac{\xi}{\sigma - 1})\alpha(1 - \lambda) + \lambda\epsilon\xi > \epsilon\xi \tag{94}$$

Then I am done. To prove Condition 94 is true, I spread the terms including spreading ϵ as $\frac{\sigma\alpha}{\sigma-1}$ and I have:

$$(1 + \frac{\xi}{\sigma - 1})(\alpha - \alpha\lambda) + \lambda\frac{\sigma\alpha}{\sigma - 1}\xi > \frac{\sigma\alpha}{\sigma - 1}\xi$$

This is the same as proving:

$$\alpha - \alpha\lambda + \frac{\alpha\xi}{\sigma - 1} - \frac{\alpha\lambda\xi}{\sigma - 1} + \lambda\frac{\sigma\alpha}{\sigma - 1}\xi - \frac{\sigma\alpha}{\sigma - 1}\xi > 0$$

Re-arrange the terms gives:

$$\alpha(1 - \lambda) + \frac{\alpha\xi}{\sigma - 1}(1 - \lambda + \lambda\sigma - \sigma) > 0$$

Re-arrange the terms further gives:

$$\begin{aligned}
\alpha(1 - \lambda) + \frac{\alpha\xi}{\sigma - 1}((1 - \lambda) + (\lambda - 1)\sigma)& \\
= \alpha(1 - \lambda) + \frac{\alpha\xi}{\sigma - 1}(1 - \lambda)(1 - \sigma)& \\
= \alpha(1 - \lambda) - \alpha\xi(1 - \lambda)& \\
= \alpha(1 - \lambda)(1 - \xi) > 0&
\end{aligned} \tag{95}$$

So I proved that Condition 93 satisfied meaning Condition 92 automatically get satisfied, which means I can order these three conditions in there strictness as: Condition 92 is the loosest, Condition 93 is in the middle, and Condition 91 is the strictest.

I Calibrated Trade Cost Matrix and Regional Aggregate Fit

J Validate welfare and N_I

I validate the welfare changes predicted by my model using BEA-reported regional real personal income (RPI) and real personal consumption expenditure (RPCE). In my static model, the trade balance condition implies that personal income is equivalent to personal consumption expenditure, and welfare changes are therefore measured by the percentage change in real personal income (equivalently, real personal expenditure). In the data, however, these two measures are not equivalent, and household welfare is affected by many factors beyond tariff policy alone.

To discipline the validation exercise, I adopt a simple measurement framework. Suppose there exists a true change in household welfare, denoted by $\widehat{\text{welfare}}^*$, which can be decomposed into a component driven by tariff policy and a component driven by all other factors:

$$\widehat{\text{welfare}}^*_S = \widehat{\text{welfare}}^*_{\text{tariff},S} + \widehat{\text{welfare}}^*_{\text{other},S} \tag{96}$$

The welfare change predicted by my model is a noisy indicator of the tariff-driven component, such that

$$\widehat{\text{welfare}}^{\text{model}}_S = \mu \widehat{\text{welfare}}^*_{\text{tariff},S} + \epsilon_S \tag{97}$$

where ϵ captures model mis-specification and aggregation error.

The data-reported real personal income and real personal consumption expenditures are noisy indicators of the true welfare change. In particular,

$$\widehat{\text{RPCE}}_S = \alpha_1 \widehat{\text{welfare}}^*_{\text{tariff},S} + \alpha_2 \widehat{\text{welfare}}^*_{\text{other},S} + \nu_S \tag{98}$$

$$\widehat{\text{RPI}}_S = \beta_1 \widehat{\text{welfare}}^*_{\text{tariff},S} + \beta_2 \widehat{\text{welfare}}^*_{\text{other},S} + \iota_S \tag{99}$$

With two noisy signals, a linear combination of $\widehat{\text{RPCE}}$ and $\widehat{\text{RPI}}$ can weakly improve the proxy for $\hat{W}^{\text{tariff}}$ relative to using either measure alone, by partially filtering out the non-tariff component $\hat{W}^{\text{other}}$.

The first column of Table 6 provides evidence consistent with the idea that combining the two indicators helps attenuate welfare changes driven by non-tariff factors, and that the model-predicted tariff-induced welfare change aligns with the corresponding data proxy[7].

In the second column, I evaluate the performance of the original Melitz (2003) framework, which abstracts from capital and capital mobility.[8] As in the first column, the coefficients on RPCE and RPI display a mirror pattern, with similar magnitudes and opposite signs. However, unlike the results from my model, both coefficients are statistically insignificant. This indicates that the original Melitz (2003) framework does not approximate tariff-induced welfare changes as effectively as the model with capital and capital mobility.

The third and fourth columns report placebo tests based on pre-policy variation. Specifically, I replace the 2017–2018 changes in RPCE and RPI with their percentage changes from 2016 to 2017, a period preceding the implementation of the tariff policy. Because no major tariff shocks occurred during this period, the model-predicted tariff-induced welfare change should not systematically align with these pre-policy income or expenditure changes. Consistent with this expectation, the coefficients in Columns (3) and (4) are statistically insignificant, providing further support that the significant results in Column (1) reflect the tariff policy shock rather than spurious correlations.

In the fifth column, I conduct a horse-race regression that jointly includes the welfare changes predicted by my model and by the Melitz (2003) framework. The coefficient on my model-predicted welfare change is positive and statistically significant, whereas the coefficient on the Melitz-predicted welfare change is small and statistically insignificant. This result indicates that, conditional on both measures, variation in regional real personal income changes is primarily explained by the welfare changes predicted by the model with capital and capital mobility[9].

Digging a bit deeper, I next validate whether the model-predicted percentage change in the number of firms aligns with the data-implied percentage change in the number of establishments following the tariff policy.

The most direct validation exercise is to test whether the model-predicted change in the number of firms, $\hat{N}_S^{\text{model}}$, is significantly positively correlated with the data-reported change, $\hat{N}_S^{\text{data}}$. Column (1) of Table 7 shows that this correlation is statistically insignificant. This null result is not surprising. Even if firm entry responds to wage changes as implied by the model, the tariff-induced component of the true wage response may be obscured by

[7]Table **??** changes the dependent variable to $\hat{W}_I - \hat{P}_I$ indicates the welfare change for the pure wage earners.

[8]I use the same calibrated parameters for this specification, except that labor intensity is set to $\xi = 1$ and the regional capital endowment shares γ_S are not used when computing the equilibrium.

[9]The validation for the model predicted number of firms can be found in Appendix J

Table 6: Tariff welfare validation: Model vs. Melitz (HC3 robust SEs)

	(1) %Δ HH welfare (my model)	(2) %Δ HH welfare (Melitz)	(3) %Δ HH welfare (my model)	(4) %Δ HH welfare (Melitz)
%Δ RPCE	-20.519***	-14.773		
	(6.148)	(14.187)		
%Δ RPCE (2016–2017)			-5.540	-17.096
			(6.560)	(15.553)
%Δ RPI	15.015**	15.241		
	(5.940)	(14.954)		
%Δ RPI (2016–2017)			-1.796	5.312
			(3.808)	(4.801)
Constant	0.057	-0.225	0.131	0.108
	(0.069)	(0.153)	(0.109)	(0.198)
Observations	51.000	51.000	51.000	51.000
R-squared	0.338	0.087	0.066	0.082
Adj. R-squared	0.310	0.049	0.027	0.044

wage movements driven by non-tariff factors. As a result, the mapping between $\hat{N}_S^{\text{model}}$ and $\hat{N}_S^{\text{data}}$ can be weak in the data despite the structural relationship holding in the model. Mathematically, this can be expressed as follows:

$$\hat{N}_S^{\text{model}} = g\Big(\hat{W}_S^{\text{model}}\Big) \;\approx\; (1-\xi)\,\hat{W}_S^{\text{model}} + \delta_S^N, \tag{100}$$

$$\hat{W}_S^{*} = \hat{W}_{\text{tariff},S}^{*} + \hat{W}_{\text{other},S}^{*}, \tag{101}$$

$$\hat{W}_S^{\text{model}} = \hat{W}_{\text{tariff},S}^{*} + \eta_S, \tag{102}$$

$$\hat{N}_S^{\text{data}} = g\Big(\hat{W}_S^{*}\Big) + \epsilon_S^N \;\approx\; (1-\xi)\,\hat{W}_S^{*} + \epsilon_S^N. \tag{103}$$

Substituting these expressions yields

$$\hat{N}_S^{\text{data}} \approx (1-\xi)\hat{W}_S^{\text{model}} + (1-\xi)\big(\hat{W}_{\text{other},S}^{*} - \eta_S\big) + \epsilon_S^N. \tag{104}$$

Therefore, even under a linear approximation, regressions of $\hat{N}_S^{\text{data}}$ on $\hat{N}_S^{\text{model}}$ or on $\hat{W}_S^{\text{model}}$ may yield weak or insignificant correlations when wage movements unrelated to tariffs and model-implied noise dominate the tariff-induced component of the true wage response, a difficulty that is further compounded when the underlying mapping is nonlinear and not invertible.

Unfortunately, unlike in the welfare case, I do not observe an alternative data proxy for the change in the number of firms that would allow me to net out non-tariff wage movements. I therefore focus on constructing a more accurate proxy for the tariff-induced component of the true wage change, $W^*_{\text{tariff},S}$, which in turn governs the model-implied response of firm entry.

From Equation 28, the model-predicted change in regional GDP is also tied to movements in the tariff-induced component of wages:

$$\widehat{\text{GDP}}_S^{\text{model}} = h\Big(\hat{W}_S^{\text{model}}\Big) \;\approx\; \hat{W}_S^{\text{model}} + \delta_S^{\text{GDP}}. \tag{105}$$

Because $\hat{N}_S^{\text{model}}$ and $\widehat{\text{GDP}}_S^{\text{model}}$ are distinct nonlinear transformations of the same underlying tariff-induced wage shock, the linear projection of $W^*_{\text{tariff},S}$ on $\big(\hat{N}_S^{\text{model}}, \widehat{\text{GDP}}_S^{\text{model}}\big)$ can be at least weakly more informative than the projection on $\hat{W}_S^{\text{model}}$ alone.

Note that this combination must exclude the direct proxy $\hat{W}_S^{\text{model}}$, since including it would lead the optimal linear combination to place all weight on $\hat{W}_S^{\text{model}}$, yielding no improvement[10].

Column (2) of Table 7 should be interpreted as a linear projection rather than a structural relationship. Since $\hat{N}_S^{\text{model}}$ and $\widehat{\text{GDP}}_S^{\text{model}}$ are both noisy, nonlinear functions of the same latent tariff-induced wage component, $\hat{W}^*_{\text{tariff},S}$, the coefficients should not be read as evidence that the model-implied change in the number of firms matches the observed change in establishments. Rather, the joint significance of the two regressors, together with the improvement in fit relative to Column (1), suggests that changes in establishments covary with the underlying tariff-induced wage response that the model is intended to capture. The opposite signs reflect partialling-out of shared variation and are not meaningful on their own.

[10]Check Appendix **??** for a detailed explanation

Table 7: Tariff Number of Firms Validation(HC3 robust SEs)

	(1)	(2)
	%Δ #Firms (data)	%Δ #Firms (data)
%Δ #firms (model)	1.651	4.619***
	(1.263)	(1.370)
%Δ GDP (model)		-1.202***
		(0.391)
Constant	-0.012	-0.024**
	(0.012)	(0.010)
Observations	51	51
R-squared	0.026	0.138
Adj. R-squared	0.006	0.102

K Mobile Capital Under Armington (1969) Setup

In this section, I show that without the heterogeneous-firm assumption, capital mobility does not generate cross-country competition for the mobile factor, capital. The main intuition is that under a homogeneous-firm setup, no fixed cost is required for firm entry. This can be interpreted as implying that no factors are "wasted"; all factors are fully used in the production of consumption goods. Because the global endowment of factors remains unchanged, total production does not change. As a result, a country's share of world sales, $\frac{\sum_I V_{JI}}{\sum_I V_{JI}+\sum_I V_{LI}}$, remains unchanged.

To illustrate this more specifically, I show the full derivation under the Armington (1969) setup. With only two countries in the Armington (1969) framework, there are two goods in the world. Consumers in both countries derive utility from consuming a composite of the two goods.

For consumers in country J, they solve the following utility maximization problem:

$$\max_{q_{JJ};q_{JL}} \left(q_{JJ}^{\frac{\sigma-1}{\sigma}} + q_{JL}^{\frac{\sigma-1}{\sigma}} \right) \quad \text{s.t.} \quad p_J q_{JJ} + (1+\tau_{JL}) p_L q_{JL} = Y_J.$$

Consumers in country J demand goods produced in country J in the quantity

$$q_{JJ} = \frac{p_J^{-\sigma}}{P_J^{1-\sigma}} Y_J$$

and goods produced in country L in the quantity

$$q_{JL} = \frac{((1+\tau_{JL})p_L)^{-\sigma}}{P_J^{1-\sigma}} Y_J.$$

Producers in country J receive demand from both countries J and L, such that

$$Q_J = \frac{p_J^{-\sigma}}{P_J^{1-\sigma}} Y_J + \frac{((1+\tau_{LJ})p_J)^{-\sigma}}{P_L^{1-\sigma}} Y_L.$$

Producers then solve a cost minimization problem to determine the factor inputs required for production. The optimal demands for labor and capital are therefore

$$L_J = \xi \frac{Q_J}{z_J} \left(\frac{w_J}{r}\right)^{\xi-1}$$

$$K_J = (1-\xi) \frac{Q_J}{z_J} \left(\frac{w_J}{r}\right)^{\xi}$$

Under monopolistic competition, firms set prices as a constant markup over marginal cost. Therefore,

$$p_J = \frac{\sigma}{\sigma-1} \frac{w_J^{\xi} r^{1-\xi}}{z_J}$$

$$p_L = \frac{\sigma}{\sigma-1} \frac{w_L^{\xi} r^{1-\xi}}{z_L}.$$

K.1 Equilibrium conditions

Thus there are two labor market clearing conditions as:

$$\frac{w_J L_J}{\xi} = \sum_I V_{JI}$$

$$\frac{w_L L_L}{\xi} = \sum_I V_{LI}$$

one capital market clearing condition:

$$\frac{r K_0}{1-\xi} = \sum_I V_{JI} + \sum_I V_{LI}$$

and one trade balance condition:

$$w_J L_J + \gamma r K_0 = V_{JJ} + V_{LJ}$$

with the trade values be in the form of:

$$V_{SI} = (\frac{W_S^{\xi} r^{1-\xi}(1+\tau_{SI})}{z_S})^{1-\sigma} B_I$$

where $B_I = \frac{w_I L_I + r K_I}{P_I^{1-\sigma}}$

K.2 Comparative Statics

Log-differentiating the equilibrium conditions gives

$$\begin{pmatrix} (1-\xi+\sigma\xi)\sum_I V_{JI} & 0 & -V_{JJ} & -V_{JL} \\ 0 & (1-\xi+\sigma\xi)\sum_I V_{LI} & -V_{LJ} & -V_{LL} \\ \sum_I V_{JI} & \sum_I V_{LI} & 0 & 0 \\ \xi(\sum_I V_{JI} - (1-\sigma)V_{JJ}) & \xi(\sigma-1)V_{LJ} & -(V_{JJ}+V_{LJ}) & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} -(\sigma-1)V_{JL}\hat{\tau}_{JL} \\ -(\sigma-1)V_{LJ}\hat{\tau}_{LJ} \\ 0 \\ -(\sigma-1)V_{LJ}\hat{\tau}_{LJ} \end{pmatrix}.$$

Under the symmetric-country assumption, denote the export share by λ. The system simplifies to

$$\begin{pmatrix} (1-\xi+\sigma\xi) & 0 & -(1-\lambda) & -\lambda \\ 0 & (1-\xi+\sigma\xi) & -\lambda & -(1-\lambda) \\ 1 & 1 & 0 & 0 \\ \xi(1-(1-\sigma)(1-\lambda)) & \xi(\sigma-1)\lambda & -1 & 0 \end{pmatrix} \begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} -(\sigma-1)\lambda\hat{\tau}_{JL} \\ -(\sigma-1)\lambda\hat{\tau}_{LJ} \\ 0 \\ -(\sigma-1)\lambda\hat{\tau}_{LJ} \end{pmatrix}.$$

K.3 Bilateral Trade Liberalization

When both countries reduce tariffs by the same amount such that $\hat{\tau}_{JL} = \hat{\tau}_{LJ} = \hat{\tau} < 0$, we obtain

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \lambda(\sigma-1)\hat{\tau} \\ \lambda(\sigma-1)\hat{\tau} \end{pmatrix},$$

which is the same result as in Section 2.

K.4 Unilateral Trade Liberalization

Suppose only the domestic country reduces its tariff, such that $\hat{\tau}_{LJ} = \hat{\tau} < 0$ and $\hat{\tau}_{JL} = 0$. The solution becomes

$$\begin{pmatrix} \hat{W}_J \\ \hat{W}_L \\ \hat{B}_J \\ \hat{B}_L \end{pmatrix} = \begin{pmatrix} \frac{\lambda\hat{\tau}}{A}(\sigma-1)(1-\lambda) \\ -\frac{\lambda\hat{\tau}}{A}(\sigma-1)(1-\lambda) \\ \frac{\lambda\hat{\tau}}{A}(\sigma-1)(1+\xi(\sigma-1)+\xi\lambda) \\ -\frac{\lambda\hat{\tau}}{A}(\sigma-1)\xi((1-2\lambda)(\sigma(1-\lambda)+\lambda)+\lambda) \end{pmatrix},$$

where

$$A = 1-\xi+2\lambda\xi(2\lambda-1+2\sigma-2\lambda\sigma).$$

Given an export share $\lambda < \frac{1}{2}$, it follows that $A > 0$.

The change in the price index is

$$\hat{P}_I = \frac{1}{\sigma-1}\big(\hat{B}_I - s_I\,\hat{W}_I\big),$$

where $s_I = \xi$ under the symmetric setup. Therefore,

$$\hat{P}_J = \frac{\lambda\hat{\tau}}{A}\big(1-\xi+\xi(\sigma-1+2\lambda)\big) < 0.$$

Thus, unilateral trade liberalization unambiguously reduces the domestic price index.

Note that

$$\hat{W}_J = \hat{V}_J = \hat{\tilde{K}}_J < 0,$$

where $\hat{\tilde{K}}_J$ denotes the amount of capital used by country J for production. Hence, unilateral liberalization leads country J to lose capital.

However, the change in domestic welfare is

$$\xi\hat{W}_J - \hat{P}_J = \frac{\lambda\hat{\tau}}{A}\big(\xi((\sigma-1)(1-\lambda)+1-(\sigma-1+2\lambda))-1\big).$$

Since

$$\xi((\sigma-1)(1-\lambda)+1-(\sigma-1+2\lambda))-1 < (\sigma-1)(1-\lambda)+1-(\sigma-1+2\lambda)-1 = -(1+\sigma)\lambda < 0,$$

we obtain

$$\xi \hat{W}_J - \hat{P}_J > 0.$$

Therefore, unilateral trade liberalization raises domestic welfare, while a unilateral tariff increase would unambiguously reduce welfare.

Compared with Section 2, this result shows that, without the extensive-margin effect, a unilateral tariff increase under the symmetric-country setup cannot generate welfare gains, even though it attracts capital inflows. Thus, the standard result that trade liberalization is welfare-improving continues to hold even in the presence of capital mobility.