ECON 113 DI

Dizhi Wang

OH This Week: Thursday 8:30am – 12:30 pm E2 403B

OLS (Ordinary Least Square) Estimate

- We CONSTRUCT it to estimate the conditional mean
- What should we ensure while CONSTRUCTING it?
- Firstly, $\bar{y} = \frac{1}{n}\sum_i y_i$

OLS (Ordinary Least Square) Estimate

- We CONSTRUCT it to estimate the conditional mean
- What should we ensure while CONSTRUCTING it?
- Firstly, $\bar{y} = \frac{1}{n}\sum_i y_i$
- What is $\sum_i (y_i - \bar{y})$?

OLS (Ordinary Least Square) Estimate

- We CONSTRUCT it to estimate the conditional mean
- What should we ensure while CONSTRUCTING it?
- Firstly, $\bar{y} = \frac{1}{n}\sum_i y_i$
- Mathematically $\sum_i (y_i - \bar{y}) = 0$
- When we are CONSTRUCTing OLS, we make sure $\sum_i (\hat{y}_i - \bar{y}) = 0$
- Property of the OLS:
  - $\bar{y} = \frac{1}{n}\sum_i y_i = \frac{1}{n}\sum_i \hat{y}_i$ => $\sum_i (\hat{y}_i - y_i) = ?$

OLS (Ordinary Least Square) Estimate

- We CONSTRUCT it to estimate the conditional mean
- What should we ensure while CONSTRUCTING it?
- Firstly, $\bar{y} = \frac{1}{n}\sum_i y_i$
- Mathematically $\sum_i (y_i - \bar{y}) = 0$
- When we are CONSTRUCTing OLS, we make sure $\sum_i (\hat{y}_i - \bar{y}) = 0$
- Property of the OLS:
  - $\bar{y} = \frac{1}{n}\sum_i y_i = \frac{1}{n}\sum_i \hat{y}_i$ => $\sum_i (y_i - \hat{y}_i) = 0 = \sum_i \hat{u}_i$
  - Data mean and fitted value mean are the same
  - Sum of residuals will always be 0

OLS (Ordinary Least Square) Estimate

- We CONSTRUCT it to estimate the conditional mean
- What should we ensure while CONSTRUCTING it?
- Secondly, we have an important assumption: $E[u|x] = 0 = E[ux]$

OLS (Ordinary Least Square) Estimate

- Given a dataset that has a column for variable y and x
- Each row is an observation of y, x denoted as x_i, y_i
- Secondly, we have an important assumption:$E[u|x] = 0 = E[ux]$
- Based on this assumption we **CONSTRUCT** OLS in the way that
  - $\sum_i (x_i \widehat{u_i}) = 0$

OLS (Ordinary Least Square) Estimate

- Given a dataset that has a column for variable y and x
- Each row is an observation of y, x denoted as x_i, y_i
- Secondly, we have an important assumption: $E[u|x] = E[ux] = 0$
- Based on this assumption we **CONSTRUCT** OLS in the way that
  - $\sum_i (x_i \widehat{u_i}) = 0$
- Property of the OLS:
  - $\bar{y} = \frac{1}{n}\sum_i y_i = \frac{1}{n}\sum_i \widehat{y_i}$ => $\sum_i (\widehat{y_i} - y_i) = 0 = \sum_i \widehat{u_i}$
  - $\sum_i (x_i \widehat{u_i}) = 0$

OLS (Ordinary Least Square) Estimate

- Property of the OLS:
  - $\sum_i (\widehat{y_i} - y_i) = 0 = \sum_i \widehat{u_i}$
  - $\sum_i x_i \, \widehat{u_i} = 0$
  - What is $\sum_i \widehat{y_i} \widehat{u_i}$?
    - $\sum_i \widehat{y_i} \widehat{u_i} = \sum_i (\widehat{\beta_0} + \widehat{\beta_1} x_i) \widehat{u_i} =$?

OLS (Ordinary Least Square) Estimate

- Property of the OLS:
  - $\sum_i (\widehat{y_i} - y_i) = 0 = \sum_i \widehat{u_i}$
  - $\sum_i x_i \, \widehat{u_i} = 0$
  - What is $\sum_i \widehat{y_i} \widehat{u_i}$?
    - $\sum_i \widehat{y_i} \widehat{u_i} = \sum_i (\widehat{\beta_0} + \widehat{\beta_1} x_i) \widehat{u_i} = \widehat{\beta_0} \sum_i \widehat{u_i} + \widehat{\beta_1} \sum_i x_i \widehat{u_i} = 0$

OLS (Ordinary Least Square) Estimate

- Property of the OLS:
  - $\sum_i (\widehat{y_i} - y_i) = 0 = \sum_i \widehat{u_i}$
    - Sum of residuals is equal to 0
  - $\sum_i x_i \, \widehat{u_i} = 0$
    - The sample covariance between the regressor and the OLS residuals is zero
  - $\sum_i \widehat{y_i} \widehat{u_i} = 0$
    - The fitted values are uncorrelated (orthogonal) to the residuals in OLS.

OLS

- -1+2+3-5=-1
- -5+5-0.5-0.5=-1
- Question: Do the two series look the same? If don’t, What makes them look differently?

OLS

- -1+2+3-5=-1
- -5+5-0.5-0.5=-1
- Mean values of the two series are the same yet variation may not be
- Every data will have its own variation:
  - $\sum_i (y_i - \bar{y})^2$: total sum of squares (SST)

OLS

- -1+2+3-5=-1
- -5+5-0.5-0.5=-1
- Mean values of the two series are the same yet variation may not be
- The data will have a variation:
  - $\sum_i (y_i - \bar{y})^2$: total sum of squares (SST)
- OLS takes care of the mean but not the variation
  - $\sum_i (\widehat{y_i} - \bar{y}) = 0$ is what OLS ensured
  - But it doesn't ensure equivalence of variation: $\sum_i (\widehat{y_i} - \bar{y})^2 \neq \sum_i (y_i - \bar{y})^2$
    - $\sum_i (\widehat{y_i} - \bar{y})^2$: explained sum of squares (SSE)
    - $SST - SSE = SSR = \sum_i \widehat{u_i}^2$

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$

```
. reg score aldummy
```

Source	SS	df	MS		
				Number of obs	= 1,000
				F(1, 998)	= 6252.75
Model	24460.0286	1	24460.0286	Prob > F	= 0.0000
Residual	3904.05974	998	3.91188351	R-squared	= 0.8624
				Adj R-squared	= 0.8622
Total	28364.0883	999	28.3924808	Root MSE	= 1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$
- Residual SS <=> SSR
  - $\sum_i \hat{u}_i{}^2 = \sum_i (\hat{y}_i - y_i)^2$

```
. reg score aldummy
```

Source	SS	df	MS		
Model	24460.0286	1	24460.0286	Number of obs	= 1,000
Residual	3904.05974	998	3.91188351	F(1, 998)	= 6252.75
				Prob > F	= 0.0000
				R-squared	= 0.8624
				Adj R-squared	= 0.8622
Total	28364.0883	999	28.3924808	Root MSE	= 1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$
- Residual SS <=> SSR
  - $\sum_i \hat{u}_i^{\,2} = \sum_i (\hat{y}_i - y_i)^2$
- Total SS <=> SST = SSE + SSR
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2$

```
. reg score aldummy
```

Source	SS	df	MS		
Model	24460.0286	1	24460.0286	Number of obs	= 1,000
Residual	3904.05974	998	3.91188351	F(1, 998)	= 6252.75
				Prob > F	= 0.0000
Total	28364.0883	999	28.3924808	R-squared	= 0.8624
				Adj R-squared	= 0.8622
				Root MSE	= 1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$
- Residual SS <=> SSR
  - $\sum_i \hat{u}_i{}^2 = \sum_i (\hat{y}_i - y_i)^2$
- Total SS <=> SST = SSE + SSR
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2$
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i + \hat{y}_i - \bar{y})^2$
  - Expand it

```
. reg score aldummy
```

Source	SS	df	MS		
Model	24460.0286	1	24460.0286	Prob > F	0.0000
Residual	3904.05974	998	3.91188351	R-squared	0.8624
				Adj R-squared	0.8622
Total	28364.0883	999	28.3924808	Root MSE	1.9778

Number of obs	=	1,000
F(1, 998)	=	6252.75

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$
- Residual SS <=> SSR
  - $\sum_i \hat{u}_i{}^2 = \sum_i (\hat{y}_i - y_i)^2$
- Total SS <=> SST = SSE + SSR
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2$
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i + \hat{y}_i - \bar{y})^2$
  - $= \sum_i (y_i - \hat{y}_i)^2 + 2\sum_i (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) + \sum_i (\hat{y}_i - \bar{y})^2$

```
. reg score aldummy
```

Source	SS	df	MS		
Model	24460.0286	1	24460.0286	Number of obs	= 1,000
Residual	3904.05974	998	3.91188351	F(1, 998)	= 6252.75
				Prob > F	= 0.0000
				R-squared	= 0.8624
				Adj R-squared	= 0.8622
Total	28364.0883	999	28.3924808	Root MSE	= 1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$
- Residual SS <=> SSR
  - $\sum_i \hat{u}_i{}^2 = \sum_i (\hat{y}_i - y_i)^2$
- Total SS <=> SST = SSE + SSR
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2$
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i + \hat{y}_i - \bar{y})^2$
  - $= \sum_i (y_i - \hat{y}_i)^2 + 2\sum_i (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) + \sum_i (\hat{y}_i - \bar{y})^2$
  - $2\sum_i (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) = 2\sum_i \hat{u}_i (\hat{y}_i - \bar{y}) = ?$

```
. reg score aldummy
```

Source	SS	df	MS	Number of obs	=	1,000
				F(1, 998)	=	6252.75
Model	24460.0286	1	24460.0286	Prob > F	=	0.0000
Residual	3904.05974	998	3.91188351	R-squared	=	0.8624
				Adj R-squared	=	0.8622
Total	28364.0883	999	28.3924808	Root MSE	=	1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- Model SS <=> SSE
  - $\sum_i (\hat{y}_i - \bar{y})^2$
- Residual SS <=> SSR
  - $\sum_i \hat{u}_i^{\,2} = \sum_i (\hat{y}_i - y_i)^2$
- Total SS <=> SST = SSE + SSR
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2$
  - $\sum_i (y_i - \bar{y})^2 = \sum_i (y_i - \hat{y}_i + \hat{y}_i - \bar{y})^2$
  - $= \sum_i (y_i - \hat{y}_i)^2 + 2\sum_i (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) + \sum_i (\hat{y}_i - \bar{y})^2$
  - $2\sum_i (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) = 2\sum_i \hat{u}_i(\hat{y}_i - \bar{y}) = 2\sum_i \hat{u}_i\hat{y}_i - 2\bar{y}\sum_i \hat{u}_i = 0$
  - $= \sum_i (y_i - \hat{y}_i)^2 + \sum_i (\hat{y}_i - \bar{y})^2$

```
. reg score aldummy

      Source |       SS       df       MS              Number of obs =     1,000
-------------+------------------------------           F(1, 998)     =  6252.75
       Model |  24460.0286     1  24460.0286            Prob > F      =  0.0000
    Residual |  3904.05974   998  3.91188351            R-squared     =  0.8624
-------------+------------------------------           Adj R-squared =  0.8622
       Total |  28364.0883   999  28.3924808            Root MSE      =  1.9778

------------------------------------------------------------------------------
       score | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    aldummy |    9.892386   .1251024    79.07   0.000     9.646892    10.13788
      _cons |    75.05864   .0890778   842.62   0.000     74.88384    75.23344
------------------------------------------------------------------------------
```

From Regression Table

- R^2<=> R-squared :
  - Coefficient of determination
    - How much the variation matches
    - $R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$
  - $0 \leq R^2 \leq 1$
    - $R^2 = 1$: Perfect fit
    - $R^2 \approx 0$: Poor fit

```
. reg score aldummy
```

Source	SS	df	MS	Number of obs	=	1,000
				F(1, 998)	=	6252.75
Model	24460.0286	1	24460.0286	Prob > F	=	0.0000
Residual	3904.05974	998	3.91188351	R-squared	=	0.8624
				Adj R-squared	=	0.8622
Total	28364.0883	999	28.3924808	Root MSE	=	1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- $y = \beta_0 * 1 + \beta_1 * x + u$
- Estimated coefficient in front of variable “aldummy”
  - $x = aldummy$
  - $\widehat{\beta_1} = 9.892386$

```
. reg score aldummy
```

Source	SS	df	MS	Number of obs	=	1,000
				F(1, 998)	=	6252.75
Model	24460.0286	1	24460.0286	Prob > F	=	0.0000
Residual	3904.05974	998	3.91188351	R-squared	=	0.8624
				Adj R-squared	=	0.8622
Total	28364.0883	999	28.3924808	Root MSE	=	1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

From Regression Table

- $y = \beta_0 * 1 + \beta_1 * x + u$
- Estimated coefficient in front of constant “1”
  - $const = 1$
  - $\widehat{\beta_0} = 75.05864$

```
. reg score aldummy
```

Source	SS	df	MS	Number of obs	=	1,000
				F(1, 998)	=	6252.75
Model	24460.0286	1	24460.0286	Prob > F	=	0.0000
Residual	3904.05974	998	3.91188351	R-squared	=	0.8624
				Adj R-squared	=	0.8622
Total	28364.0883	999	28.3924808	Root MSE	=	1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

Interpretation

- The predicted/fitted mean of score when aldummy = 0 (the baseline group) is 75.05864
- The difference in mean score between the group with aldummy = 1 and the group with aldummy = 0 is 9.892386

```
. reg score aldummy
```

Source	SS	df	MS	Number of obs	=	1,000
				F(1, 998)	=	6252.75
Model	24460.0286	1	24460.0286	Prob > F	=	0.0000
Residual	3904.05974	998	3.91188351	R-squared	=	0.8624
				Adj R-squared	=	0.8622
Total	28364.0883	999	28.3924808	Root MSE	=	1.9778

score	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
aldummy	9.892386	.1251024	79.07	0.000	9.646892	10.13788
_cons	75.05864	.0890778	842.62	0.000	74.88384	75.23344

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
  - *Note:* it is u_i not $\widehat{u_i}$ here
  - Only u_i is rv
  - x_i and $\bar{x}$ are all known values from data
  - We can treat $(x_i - \bar{x})$ as a coefficient

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
- $E[\widehat{\beta_1}] = \beta_1 + \frac{E(\sum_i (x_i - \bar{x}) u_i)}{\sum_i (x_i - \bar{x})^2}$
  - $E(\sum_i (x_i - \bar{x}) u_i) = ?$
  - Hint: $E[\sum_{i=1}^{2} \alpha_i X_i] = E[\alpha_1 X_1 + \alpha_2 X_2] = \alpha_1 E[X_1] + \alpha_2 E[X_2] = \sum_i \alpha_i E[X_i]$

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
  - u_i is changing based on the change of estimation/observation
  - Only u_i is rv
- $E[\widehat{\beta_1}] = \beta_1 + \frac{E(\sum_i (x_i - \bar{x}) u_i)}{\sum_i (x_i - \bar{x})^2}$
  - $E(\sum_i (x_i - \bar{x}) u_i) = \sum_i (x_i - \bar{x}) E[u_i]$
  - With the OLS assumption $E[u_i | x_i] = 0$, we have $E[u_i] = 0$

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
  - Only u_i is rv
- $Var[\widehat{\beta_1}] = ?$
  - Hint: $Var[aX_1 + bX_2] = a^2 Var[X_1] + b^2 Var[X_2]$

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
  - Only u_i is rv
- $Var[\widehat{\beta_1}] = Var(\beta_1) + \frac{Var(\sum_i (x_i - \bar{x}) u_i)}{\left(\sum_i (x_i - \bar{x})^2\right)^2} = 0 + \frac{Var(\sum_i (x_i - \bar{x}) u_i)}{\left(\sum_i (x_i - \bar{x})^2\right)^2}$
  - $Var(\sum_i (x_i - \bar{x}) u_i) =$?

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
  - Only u_i is rv
- $Var[\widehat{\beta_1}] = Var(\beta_1) + \frac{Var(\sum_i (x_i - \bar{x}) u_i)}{\left(\sum_i (x_i - \bar{x})^2\right)^2} = 0 + \frac{Var(\sum_i (x_i - \bar{x}) u_i)}{\left(\sum_i (x_i - \bar{x})^2\right)^2}$
  - $Var(\sum_i (x_i - \bar{x}) u_i) = \sum_i (x_i - \bar{x})^2 \, Var(u_i)$

Decomposition

- $y = \beta_0 * 1 + \beta_1 * x + u$
- β_0 and β_1 are fixed values
- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are the estimations/observations of the values, which can form a distribution
- $\widehat{\beta_1} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$
  - Only u_i is rv
- $Var[\widehat{\beta_1}] = Var(\beta_1) + \frac{Var(\sum_i (x_i - \bar{x}) u_i)}{\left(\sum_i (x_i - \bar{x})^2\right)^2} = 0 + \frac{Var(\sum_i (x_i - \bar{x}) u_i)}{\left(\sum_i (x_i - \bar{x})^2\right)^2}$
  - $Var(\sum_i (x_i - \bar{x}) u_i) = \sum_i (x_i - \bar{x})^2 \, Var(u_i)$
  - $Var[\widehat{\beta_1}] = \frac{\sum_i (x_i - \bar{x})^2}{\left(\sum_i (x_i - \bar{x})^2\right)^2} Var(u_i) = \frac{Var(u_i)}{\sum_i (x_i - \bar{x})^2}$

Non-linear Interpretation

TABLE 2.3 Summary of Functional Forms Involving Logarithms

Model	Dependent Variable	Independent Variable	Interpretation of β_1
Level-level	y	x	$\Delta y = \beta_1 \Delta x$
Level-log	y	$\log(x)$	$\Delta y = (\beta_1/100)\%\Delta x$
Log-level	$\log(y)$	x	$\%\Delta y = (100\beta_1)\Delta x$
Log-log	$\log(y)$	$\log(x)$	$\%\Delta y = \beta_1 \%\Delta x$

A one-unit increase in x changes y by β_1 units.

Example:
Each additional year of education increases annual income by **$2,000** on average. ($\beta_1 = 2000$)

Non-linear Interpretation

TABLE 2.3 Summary of Functional Forms Involving Logarithms

Model	Dependent Variable	Independent Variable	Interpretation of β_1
Level-level	y	x	$\Delta y = \beta_1 \Delta x$
Level-log	y	$\log(x)$	$\Delta y = (\beta_1/100)\%\Delta x$
Log-level	$\log(y)$	x	$\%\Delta y = (100\beta_1)\Delta x$
Log-log	$\log(y)$	$\log(x)$	$\%\Delta y = \beta_1 \%\Delta x$

A 1% increase in x changes y by $\beta_1/100$ units.

Example:
A 1% increase in firm size raises total revenue by **$50** on average.($\beta_1 = 5{,}000$)

Non-linear Interpretation

TABLE 2.3 Summary of Functional Forms Involving Logarithms

Model	Dependent Variable	Independent Variable	Interpretation of β_1
Level-level	y	x	$\Delta y = \beta_1 \Delta x$
Level-log	y	$\log(x)$	$\Delta y = (\beta_1/100)\%\Delta x$
Log-level	$\log(y)$	x	$\%\Delta y = (100\beta_1)\Delta x$
Log-log	$\log(y)$	$\log(x)$	$\%\Delta y = \beta_1 \%\Delta x$

A one-unit increase in x changes y by approximately $100 \times \beta_1 \%$

Example:
Each additional year of education increases wages by **5%** on average. ($\beta_1 = 0.05$)

Non-linear Interpretation

TABLE 2.3 Summary of Functional Forms Involving Logarithms

Model	Dependent Variable	Independent Variable	Interpretation of β_1
Level-level	y	x	$\Delta y = \beta_1 \Delta x$
Level-log	y	$\log(x)$	$\Delta y = (\beta_1/100)\%\Delta x$
Log-level	$\log(y)$	x	$\%\Delta y = (100\beta_1)\Delta x$
Log-log	$\log(y)$	$\log(x)$	$\%\Delta y = \beta_1 \%\Delta x$

A 1% increase in x changes y by $\beta_1\%$

Example:
A 1% increase in advertising spending increases sales by **0.3%**. ($\beta_1 = 0.3$)