ECON 113 DI A/B

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OH : 8:30 am – 10:30 am E2 403B

Hypothesis Testing

- We estimated a coefficient, $\widehat{\beta_1}$
  - But the estimated values has **0** probability to be equal to the true β_1
- So If the true $\beta_1 = 0$, we will never get $\widehat{\beta_1} = 0$
- Given that we will mostly always get $\widehat{\beta_1} \neq 0$
- How far is $\hat{\beta}_1$ from 0 just due to random sampling noise?

Hypothesis Testing

- H_0:$\beta_1 = 0 \Rightarrow$ No effect
- H_1: $\beta_1 \neq 0 \Rightarrow$ Has some effect

- Hypothesis Testing:
  - T-statistic / p-value view
    - If the true effect were actually 0, how likely is it to see an estimate as extreme as our $\widehat{\beta_1}$ just due to random sampling noise?
  - Confidence Interval view
    - Given our estimation of $\widehat{\beta_1}$, how plausible is that the true $\beta_1 = 0$?

Hypothesis Testing

- Notice that this question can be phrased more generally

- We have an estimate $\hat{\theta}$ of the true value θ
  - H_0:$\theta = const = \theta_0$
  - H_1: $\theta \neq const = \theta_0$

- T-statistic / p-value view
  - If the true $\theta = \theta_0$, , how likely is it to see an estimate $\hat{\theta}$ being this far from θ_0?

- Confidence interval view
  - Given the estimate $\hat{\theta}$. how plausible is that the true $\theta = \theta_0$?

Hypothesis Testing

- Suppose we care about the average height of 14 years old teenagers within U.S
  - Random variable:
    - $\theta \Rightarrow$ 14 years old teenagers height in U.S
  - $\theta \sim N(\mu_\theta, \sigma_\theta^2)$
  - $Var(\theta) = E[\theta^2] - (E[\theta])^2 = E[\theta^2] - \mu_\theta^2 = \sigma_\theta^2$
  - We care about μ_θ

Hypothesis Testing

- Suppose we care about the average height of 14 years old teenagers within U.S
  - $\theta \Rightarrow$ 14 years old teenagers height
  - $\theta \sim N(\mu_\theta, \sigma_\theta^2)$
  - We care about μ_θ
- We observe the samples of the height of 14 years old teenagers within U.S
  - 1st sample: teenagers whose birth date ends with 1
    - Calculate $\hat{\theta}_{sample1} = \frac{1}{n_1}\sum_i \theta_i = \hat{\mu}_{\theta,sample1}$
  - 2nd sample: teenagers whose birth date ends with 2
    - Calculate $\hat{\theta}_{sample2} = \frac{1}{n_2}\sum \theta_i = \hat{\mu}_{\theta,sample2}$

Hypothesis Testing

- We observe the samples of the height of 14 years old teenagers within U.S
  - Different sample gives out a different $\hat{\mu} = \frac{1}{n}\sum_i \theta_i$
    - **Which forms a distribution so $\hat{\boldsymbol{\mu}} \sim \boldsymbol{N}(\boldsymbol{\mu_\theta}, \frac{\boldsymbol{\sigma_\theta^2}}{\boldsymbol{n}})$**
    - How do we know it is $\boldsymbol{N}(\boldsymbol{\mu_\theta}, \frac{\boldsymbol{\sigma_\theta^2}}{\boldsymbol{n}})$?

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      1) Normality: Central Limit Theorem

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    - How do we know it is $\boldsymbol{N(\mu_\theta, \frac{\sigma_\theta^2}{n})}$?
      1) Normality: Central Limit Theorem
      2) $E[\hat{\mu}] = E\left[\frac{1}{n}\sum_i \theta_i\right] = \frac{1}{n}\sum_i E[\theta_i] = \mu_\theta$

Hypothesis Testing

- We observe the samples of the height of 14 years old teenagers within U.S
  - Different sample gives out a different $\hat{\mu}_{\theta}$
    - Which forms a distribution so $\hat{\mu}_{\theta} \sim N(\mu_{\theta}, \frac{\sigma_{\theta}^2}{n})$
    - How do we know it is $\boldsymbol{N(\mu_{\theta}, \frac{\sigma_{\theta}^2}{n})}$?
      1) Normality: Central Limit Theorem
      2) $E[\hat{\mu}] = E\left[\frac{1}{n}\sum_i y_i\right] = \frac{1}{n}\sum_i E[y_i] = \mu$
      3) $Var[\hat{\mu}] = E[\hat{\mu}^2] - (E[\hat{\mu}])^2$

Hypothesis Testing

- We observe the samples of the height of 14 years old teenagers within U.S
  - Different sample gives out a different $\hat{\mu}_{\theta}$
    - Which forms a distribution so $\hat{\mu}_{\theta} \sim N(\mu_{\theta}, \frac{\sigma_{\theta}^2}{n})$
    - How do we know it is $\boldsymbol{N(\mu_{\theta}, \frac{\sigma_{\theta}^2}{n})}$?
      1) Normality: Central Limit Theorem
      2) $E[\hat{\mu}] = E\left[\frac{1}{n}\sum_i \theta_i\right] = \frac{1}{n}\sum_i E[\theta_i] = \mu_{\theta}$
      3) $Var[\hat{\mu}] = E[\hat{\mu}^2] - (E[\hat{\mu}])^2 = E[\left(\frac{1}{n}\sum_i \theta_i\right)^2] - \mu_{\theta}^2$

Hypothesis Testing

- We observe the samples of the height of 14 years old teenagers within U.S
  - Different sample gives out a different $\hat{\mu}_{\theta}$
    - Which forms a distribution so $\hat{\mu}_{\theta} \sim N(\mu_{\theta}, \frac{\sigma_{\theta}^2}{n})$
    - How do we know this?
      1) Normality: Central Limit Theorem
      2) $E[\hat{\mu}] = E\left[\frac{1}{n}\sum_i \theta_i\right] = \frac{1}{n}\sum_i E[\theta_i] = \mu_{\theta}$
      3) $Var[\hat{\mu}] = E[\hat{\mu}^2] - (E[\hat{\mu}])^2 = E[\left(\frac{1}{n}\sum_i \theta_i\right)^2] - \mu_{\theta}^2$
         - $E\left[\left(\frac{1}{n}\sum_i \theta_i\right)^2\right] = E[\frac{1}{n^2}(\sum_i \theta_i^2 + 2\sum_{i<j} \theta_i\theta_j)] = \frac{1}{n^2}(\sum_i E[\theta_i^2] + 2\sum_{i<j} E[\theta_i\theta_j])$

Hypothesis Testing

- We observe the samples of the height of 14 years old teenagers within U.S
  - Different sample gives out a different $\hat{\mu}$
    - Which forms a distribution so $\hat{\mu} \sim N(\mu_\theta, \frac{\sigma_\theta^2}{n})$
    - $E[\hat{\mu}] = E\left[\frac{1}{n}\sum_i \theta_i\right] = \frac{1}{n}\sum_i E[\theta_i] = \mu_\theta$
    - $Var[\hat{\mu}] = \frac{1}{n^2}\left(\sum_i E\left[\theta_i^2\right] + 2\sum_{i<j} E[\theta_i\theta_j]\right) - \mu_\theta^2$

    - **If $\boldsymbol{\theta_i}$ and $\boldsymbol{\theta_j}$ are i.i.d** (Independent and identically distributed)
      - **Independent**: Amy's height does not *cause* or *determine* Bob's height
      - **Identically distributed**: They come from the *same* population distribution of heights for 14-year-old
      - $E\left[\theta_i\theta_j\right] = E[\theta_i]E\left[\theta_j\right] = \mu_\theta^2$

Hypothesis Testing

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      - $E\left[\theta_i\theta_j\right] = E[\theta_i]E\left[\theta_j\right] = \mu_\theta^2$

    - # pairs of i, j where $i < j$ is $\frac{n(n-1)}{2}$
    - $Var[\hat{\mu}] = \frac{1}{n^2}\left(n\left(\mu_\theta^2 + \sigma_\theta^2\right) + 2\frac{n(n-1)}{2}\mu_\theta^2\right) - \mu_\theta^2 = \frac{1}{n^2}\left(n\mu_\theta^2 + n\sigma_\theta^2 + (n^2 - n)\mu_\theta^2\right) - \mu_\theta^2$
    - $Var[\hat{\mu}] = \frac{1}{n^2}\left(n\sigma_\theta^2 + n^2\mu_\theta^2\right) - \mu_\theta^2 = \frac{\sigma_\theta^2}{n}$

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    - For normal distribution, we can always normalize/standardize it

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  - Different sample gives out a different $\hat{\mu}$
    - Which forms a distribution so $\hat{\mu} \sim N(\mu_\theta, \frac{\sigma_\theta^2}{n})$
    - We can normalize/standardize the specific normal distribution for $\hat{\mu}$ by
      - Creating another rv based on $\hat{\mu}$

$$\boldsymbol{z = \frac{\hat{\mu} - \mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}}}$$

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        - Scaling and shifting a normal distribution always produces another normal distribution.
        - So we know z follows a normal distribution, but what is the new mean and variance?

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$$\boldsymbol{z = \frac{\hat{\mu} - \mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}}}$$

      - $E[z] = E\left[\frac{\hat{\mu}-\mu_\theta}{\frac{\sigma}{\sqrt{n}}}\right] = \frac{1}{\frac{\sigma}{\sqrt{n}}}(E[\hat{\mu}] - \mu_\theta) = 0$

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$$\boldsymbol{z = \frac{\hat{\mu} - \mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}}}$$

      - $E[z] = 0$
      - $Var[z] = E[z^2] - (E[z])^2$

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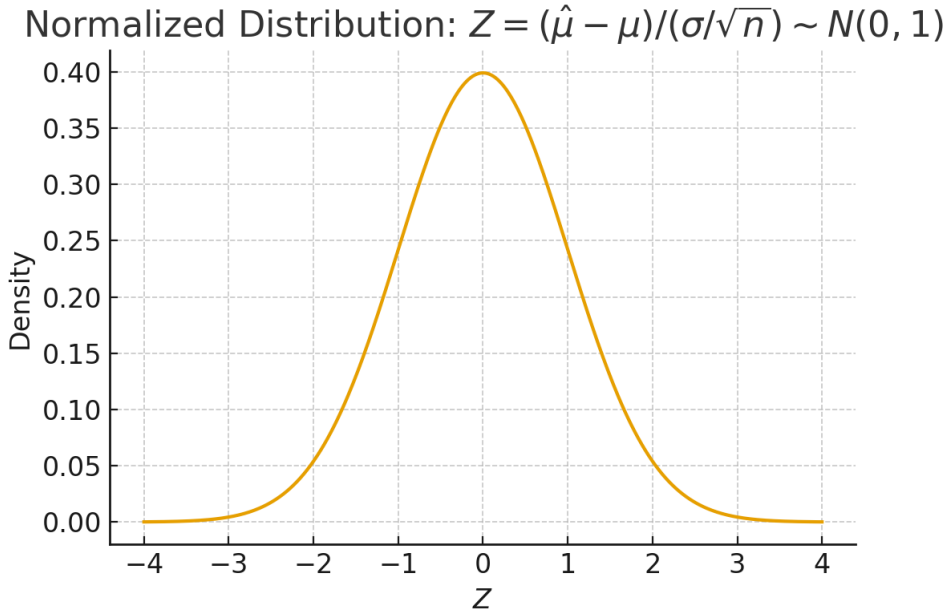
      - $E[z] = 0$
      - $Var[z] = E[z^2] = E\left[\left(\frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}}\right)^2\right]$
      - $Var[z] = E\left[\frac{1}{\frac{\sigma^2}{n^2}}\left(\hat{\mu}^2 - 2\hat{\mu}\mu_\theta + \mu_\theta^2\right)\right] = \frac{1}{\sigma^2/n^2}\left(E[\hat{\mu}^2] - 2\mu_\theta E[\hat{\mu}] + \mu_\theta^2\right)$

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    - We can normalize/standardize the specific normal distribution for $\hat{\mu}$ by
      - Creating another rv based on $\hat{\mu}$
      - $z = \frac{\hat{\mu}-\mu}{\frac{\sigma}{\sqrt{n}}}$
      - $E[z] = E\left[\frac{\hat{\mu}-\mu}{\frac{\sigma}{\sqrt{n}}}\right] = \frac{1}{\frac{\sigma}{\sqrt{n}}}(E[\hat{\mu}] - \mu) = 0$
      - $Var[z] = E[z^2] - (E[z])^2 = E[z^2] = E\left[\left(\frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}}\right)^2\right]$
      - $Var[z] = E\left[\frac{1}{\frac{\sigma^2}{n}}\left(\hat{\mu}^2 - 2\hat{\mu}\mu_\theta + \mu_\theta^2\right)\right] = \frac{1}{\sigma^2/n}\left(E[\hat{\mu}^2] - 2\mu_\theta E[\hat{\mu}] + \mu_\theta^2\right)$
      - $Var[z] = \frac{n}{\sigma^2}\left(E[\hat{\mu}^2] - 2\mu_\theta^2 + \mu_\theta^2\right) = \frac{n}{\sigma^2}\left(E[\hat{\mu}^2] - \mu_\theta^2\right) = \frac{n}{\sigma^2}Var(\hat{\mu}) = 1$
      - So $z \sim N(0,1)$

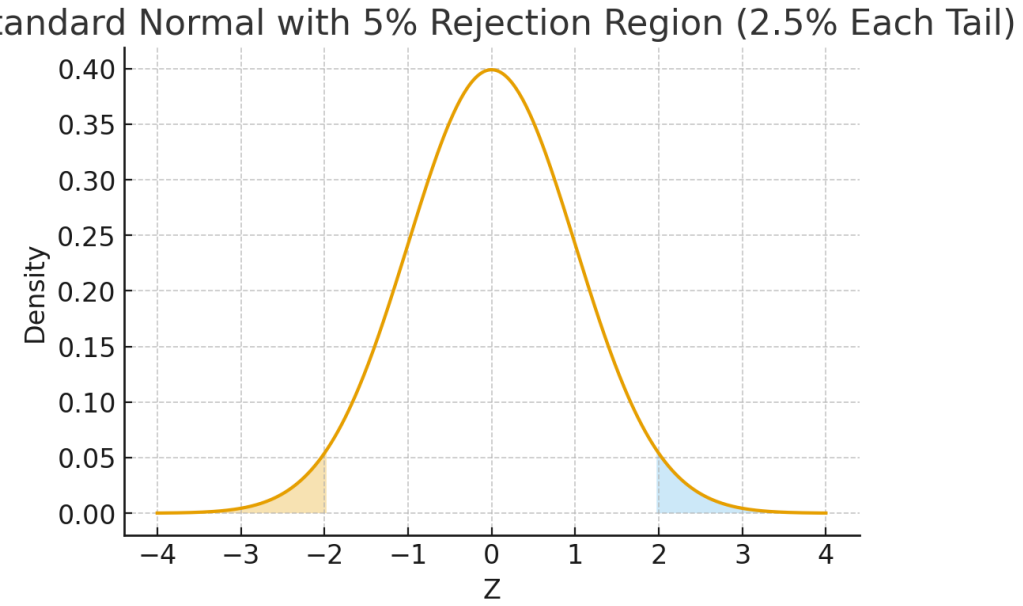
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    - We can normalize/standardize $\hat{\mu}$ such that $\frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} \sim N(0,1)$



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- Different sample gives out a different $\hat{\mu}$
  - Which forms a distribution so $\hat{\mu} \sim N(\mu, \frac{\sigma^2}{n})$
  - We can normalize/standardize $\hat{\mu}$ such that $z = \frac{\hat{\mu}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$



  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr\left(-1.96 < \frac{\hat{\mu}-\mu}{\frac{\sigma}{\sqrt{n}}} < 1.96\right) = 0.95$

Hypothesis Testing

- Different sample gives out a different $\hat{\mu}$
  - Across many samples, $\hat{\mu}$ has a distribution $\hat{\mu} \sim N(\mu_\theta, \frac{\sigma_\theta^2}{n})$
  - We can normalize/standardize $\hat{\mu}$ such that $\mathrm{z} = \frac{\hat{\mu} - \mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} \sim N(0,1)$
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  - $\Pr(-1.96 < z < 1.96) = 0.95$

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  - z is called $Z\ score$

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  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr(-1.96 < z < 1.96) = 0.95$
  - z is called $Z\ score$
  - So given the distribution of $\hat{\mu}$
  - $\Pr\left(\mu_\theta - 1.96 \times \frac{\sigma_\theta}{\sqrt{n}} < \hat{\mu} < \mu_\theta + 1.96 \times \frac{\sigma_\theta}{\sqrt{n}}\right) = 0.95$

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  - $\Pr\left(\mu_\theta - 1.96 \times \frac{\sigma_\theta}{\sqrt{n}} < \hat{\mu} < \mu_\theta + 1.96 \times \frac{\sigma_\theta}{\sqrt{n}}\right) = 0.95$
  - If we took 10,000 independent samples of size n and computed $\hat{\mu}$ for each sample,
  - then **about 95% of those 10,000 sample means** would fall inside the interval
    - $[-1.96 \times \frac{\sigma}{\sqrt{n}} + \mu, 1.96 \times \frac{\sigma}{\sqrt{n}} + \mu]$
    - Warning: this is not confidence interval

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  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr(-1.96 < z < 1.96) = 0.95$
  - z is called $Z\ score$
  - But most of the time we only have 1 sample
    - $\frac{1}{n}\sum_i \mu_i = \mu_\theta$
    - We can only get the estimation of $\frac{\sigma_\theta^2}{n}$ which is $\frac{\hat{\sigma}_\theta^2}{n}$

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- Different sample gives out a different $\hat{\mu}$
  - Across many samples, $\hat{\mu}$ has a distribution $\hat{\mu} \sim N\left(\mu_\theta, \frac{\sigma_\theta^2}{n}\right)$
  - We can normalize/standardize $\hat{\mu}$ such that $\mathrm{z} = \frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} \sim N(0,1)$
  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr\left(-1.96 < \frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} < 1.96\right) = 0.95$
  - $\frac{\hat{\mu}-\mu_\theta}{\frac{\sigma}{\sqrt{n}}}$ is called $Z\ score$
  - Instead of $\frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}}$, what we can get is
    - $\frac{\hat{\mu}-\mu_\theta}{\frac{\hat{\sigma}_\theta}{\sqrt{n}}}$ called: **t-statistics**
    - **Moreover, $\frac{\hat{\mu}-\mu_\theta}{\frac{\hat{\sigma}_\theta}{\sqrt{n}}}$ doesn't follow normal distribution ⇒ critical value is no longer 1.96**
    - **T-table in need**

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  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr\left(-1.96 < \frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} < 1.96\right) = 0.95$
  - $\Pr\left(-t_{0.975,n-1} < \frac{\hat{\mu}-\mu_\theta}{\frac{\hat{\sigma}_\theta}{\sqrt{n}}} < t_{0.975,n-1}\right) \approx 0.95$
    - If $\mu = \mu_\theta$, t-statistic will fall between $-t_{0.975,n-1}$ and $t_{0.975,n-1}$ about 95% of the time.
      - If your t-statistic fall between the interval
      - The observed t-value is not unusual under the null hypothesis $H_0: \mu = \mu_\theta$.
      - So we do *not* reject H_0.
      - T-statistic way of hypothesis testing

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  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr\left(-1.96 < \frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} < 1.96\right) = 0.95$
  - $\Pr\left(-t_{0.975,n-1} < \frac{\hat{\mu}-\mu_\theta}{\frac{\hat{\sigma}_\theta}{\sqrt{n}}} < t_{0.975,n-1}\right) \approx 0.95$
  - $\Pr\left(\hat{\mu} - t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}} < \mu_\theta < \hat{\mu} + t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}}\right) \approx 0.95$

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  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr\left(-1.96 < \frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} < 1.96\right) = 0.95$
  - $\Pr\left(-t_{0.975,n-1} < \frac{\hat{\mu}-\mu_\theta}{\frac{\hat{\sigma}_\theta}{\sqrt{n}}} < t_{0.975,n-1}\right) \approx 0.95$
  - $\Pr\left(\hat{\mu} - t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}} < \mu_\theta < \hat{\mu} + t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}}\right) \approx 0.95$
  - The confidence interval for μ_θ given the estimated $\hat{\mu}$ is
    - $[\hat{\mu} - t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}, \hat{\mu} + t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}]$

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- Different sample gives out a different $\hat{\mu}$
  - Across many samples,$\hat{\mu}$ has a distribution $\hat{\mu} \sim N\left(\mu_\theta, \frac{\sigma_\theta^2}{n}\right)$
  - We can normalize/standardize $\hat{\mu}$ such that $\mathrm{z} = \frac{\hat{\mu}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$
  - The probability of rv z being observed as bigger than 1.96 or smaller than -1.96 is 5%
  - $\Pr\left(-1.96 < \frac{\hat{\mu}-\mu_\theta}{\frac{\sigma_\theta}{\sqrt{n}}} < 1.96\right) = 0.95$
  - $\Pr\left(-t_{0.975,n-1} < \frac{\hat{\mu}-\mu_\theta}{\frac{\hat{\sigma}_\theta}{\sqrt{n}}} < t_{0.975,n-1}\right) \approx 0.95$
  - $\Pr\left(\hat{\mu} - t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}} < \mu_\theta < \hat{\mu} + t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}}\right) \approx 0.95$
  - The confidence interval for μ_θ given the estimated $\hat{\mu}$ is
    - $[\hat{\mu} - t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}, \hat{\mu} + t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}]$
    - Given $\hat{\mu}$, the plausible values for μ_θ are all values that are in $[\hat{\mu} - t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}, t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}]$

Hypothesis Testing

- Different sample gives out a different $\hat{\mu}$
  - Across many samples,$\hat{\mu}$ has a distribution $\hat{\mu} \sim N\left(\mu_\theta, \frac{\sigma_\theta^2}{n}\right)$
  - $\Pr\left(\hat{\mu} - t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}} < \mu < \hat{\mu} + t_{0.975,n-1}\frac{\hat{\sigma}_\theta}{\sqrt{n}}\right) \approx 0.95$
  - The confidence interval for μ given the estimated $\hat{\mu}$ is
    - $[\hat{\mu} - t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}, \hat{\mu} + t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}]$
  - If $\mu_\theta \in [\hat{\mu} - t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}, \hat{\mu} + t_{0.975,n-1} \times \frac{\hat{\sigma}_\theta}{\sqrt{n}}]$
  - It means that
    - μ_θ is a plausible value for the true mean, given the estimation.
    - Which means the $H_0: \mu = \mu_\theta$ is plausible
    - So we can't reject the null hypothesis
  - CI way of doing hypothesis testing

Hypothesis Testing

- Different sample gives out a different $\widehat{\boldsymbol{\beta_1}}$
  - Across many samples, $\widehat{\beta_1}$ has a distribution $\widehat{\beta_1} \sim N\left(\beta_1, \frac{\sigma^2}{\sum_i (x_i - \bar{x})^2}\right) = N(\beta_1, V(\widehat{\beta_1}))$
  - We can normalize/standardize $\widehat{\beta_1}$ such that $z = \frac{\widehat{\beta_1} - 0}{\sqrt{V(\widehat{\beta_1})}} \sim N(0,1)$
    - $\Pr(-1.96 < z < 1.96) = 0.95$
  - Can only get $\hat{V}(\widehat{\beta_1})$ instead of $V(\widehat{\beta_1})$
  - $t = \frac{\widehat{\beta_1} - 0}{\sqrt{\hat{V}(\widehat{\beta_1})}} = \frac{\widehat{\beta_1} - 0}{se(\widehat{\beta_1})}$
  - $\Pr\left(-t_{0.975,df} < t < t_{0.975,df}\right) \approx 0.95$
  - If your t-statistic fall out of the interval, e.g $t > t_{0.975,n-1}$
    - If $H_0: \beta_1 = 0$ is true
      - we would observe a t-statistic this large (or larger) less than 5% of the time.
      - so having $\widehat{\beta_1}$ is rare
      - We reject the null hypothesis

Hypothesis Testing

- Different sample gives out a different $\widehat{\beta_1}$
  - Across many samples, $\widehat{\beta_1}$ has a distribution $\widehat{\beta_1} \sim N\left(\beta_1, \frac{\hat{\sigma}^2}{\sum_i (x_i - \bar{x})^2}\right) = N\left(\beta_1, se\left(\widehat{\beta_1}\right)^2\right)$
  - From the view of confidence interval:

$$\Pr\left(\widehat{\beta_1} - t_{0.975,df} se(\widehat{\beta_1}) < \beta_1 < \widehat{\beta_1} + t_{0.975,df} se(\widehat{\beta_1})\right) \approx 0.95$$

  - The 95% confidence interval for β_1 given the estimated $\widehat{\beta_1}$ is
    - $[\widehat{\beta_1} - t_{0.975,df} se(\widehat{\beta_1}), \widehat{\beta_1} + t_{0.975,df} se(\widehat{\beta_1})]$
    - Given $\widehat{\beta_1}$, the plausible values of β_1 are all values that are within the interval
  - If $0 \in [\widehat{\beta_1} - t_{0.975,df} se(\widehat{\beta_1}), \widehat{\beta_1} + t_{0.975,df} se(\widehat{\beta_1})]$
  - It means that
    - The hypothesis that $\beta_1 = 0$ is a plausible
    - We can't reject the null hypothesis
    - CI way of doing hypothesis testing